The Relation Between Volatility and Maturity in Futures Contracts

Katherine Dusak Miller

Some students of futures markets believe that the volatility of futures prices increases as the futures contract nears maturity (see Telser, 1956; Segall, 1956; and Samuelson, 1965).

Samuelson offers an explanation for the existence of the variability effect as reviewed in Section 1 of the paper. His hypothesis about the behavior of futures prices requires that the stochastic process characterizing spot prices must be of a particular kind; other processes will yield different relations between the variability and maturity of futures contracts. Section 2 of the paper examines the behavior of a sample of futures returns for existence of a systematic volatility effect. Evidence, in this particular case, supports the view that such an effect exists. Section 3 examines the underlying spot-return series to see if its properties are consistent with the empirical results for the sample of futures returns examined in Section 2. Although the process generating the spot returns is consistent with the implications of Samuelson's hypothesis, there is a discrepancy between the observed pattern of futures return volatility and the pattern implied by the Samuelson relationship. Whether or not sampling variability could account for the discrepancy is discussed briefly at the end of the paper.

SECTION 1: RETURN VOLATILITY AS A FUNCTION OF CONTRACT MATURITY

As part of his demonstration that return volatility is related to contract maturity, Samuelson postulated that futures prices follow a martingale or "fair game" process. Mathematically, the martingale property can be represented as:

\[ Y(T, t) = E_t[X(t + T)] \]  

(1)

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where \( Y \) refers to futures prices, \( X \) to spot prices, \( T \) to the number of periods in which the futures contract will be outstanding, and \( t \) to the present time. Equation 1 states that the current futures price is equal to the spot price expected to prevail \( T \) periods hence.

The martingale property implies only that the first moment of the distribution of expected price changes is zero and says nothing about the second moment of the distribution. As Samuelson points out:

Although the sequence \( \{\Delta Y(T - n, t + n)\} \) has a zero first moment at all time periods \( T - n \), there is no reason to suppose that the riskiness of holding a futures — in the sense of the second moment or variance as measured by \( E[\{\Delta Y(T - n, t + n)\}^2] \) — should be the same when \( T \) is large and the terminal date far away as when \( T - n \) is small and the futures contract about to expire. A well-known rule of thumb is that nearness to expiration date involves greater variability of riskiness per hour or per day or per month than does farness. . . . However, the present theory can contribute an elegant explanation of why we should expect far-distant futures to move more sluggishly than near ones [1965, pp.44-45].

The elegant explanation to which Samuelson refers assumes that futures prices follow a pure martingale but that spot prices are generated by the autoregressive model:

\[
X_{t+1} = \alpha X_t + \{u_t\}
\]

(2)

where \( \alpha \) is assumed to be less than 1 and where \( X_{t+1} \) is the spot price that will prevail at time \( t + 1 \), \( X_t \) is the current spot price and \( \{u_t\} \) is a sequence of independent random drawings from a given distribution with mean 0 and variance \( \sigma^2(u) \).\(^3\)

If futures prices follow a martingale, the expected return from buying a \( T \) period futures contract at the beginning of time \( t \) and selling it one period later at the beginning of time \( t + 1 \) should be zero.

Such a return may be represented as:

\[
R(t, T) = E_{t+1}[X(t + T)] - E_t[X(t + T)].
\]

(3)

\(^3\) The assumption that spot prices follow an autoregressive process would seem to imply that the spot-commodity market is inefficient. For many commodities, however, price changes do not measure the investor's total return; hence, behavior of the price series by itself implies nothing about market efficiency. For commodities such as live cattle, total return reflects both weight gain and price change over the holding period. If, for example, the relevant holding period is one month and the price of cattle is $20 per hundredweight at the beginning of the month as well as at the end of the month, the return computed simply as the price change would be zero. Allowing for normal weight gain of two pounds a day, the total dollar return taking weight gain into account would be $12 and not zero. Thus, without knowing the pattern of weight gain over a measurement interval, one cannot point to the existence of trends in the spot-price series as a priori evidence of spot-market inefficiency.
Equation 3 represents the forecast revision of the spot price expected to prevail at the time \( t + T \) that is prompted by the receipt of new information occurring in the interval \( t \) to \( t + 1 \).

Since a specific process was assumed to be generating spot prices, \( E_{t+1}[X(t + T)] \) and \( E_t[X(t+T)] \) can be expressed in terms of the current spot price \( X_t \) and a sequence of disturbance terms \( \{u_t\} \). It can be shown then that \( R(t, T) = \sigma^2 u_{t+1} \). Of course \( E[R(t, T)] = 0 \) which must be the case if futures prices follow a martingale.

Variance of return may be expressed as:

\[
\sigma^2(R) = E[R(t, T) - E(R(t, T))]^2 = E[\alpha^T u_{t+1}]^2 = \sigma^2 \sigma^2(\alpha)
\]

Thus variance is a function of \( T \), the contract term to maturity. Since \( \alpha \) was assumed to be less than one, variance will vary inversely with \( T \).

The martingale property, coupled with the assumption that spot prices are generated by a mean-reverting process, ensures that longer-term assets will exhibit less variance than shorter-term assets. Intuitively, Samuelson's law of increasing volatility may be explained as follows: the assumption that futures prices follow a martingale means that the futures price is the expected spot price, and the assumption of a mean-reverting spot process implies, in turn, that the longer the life of the asset, the spot price will revert to its mean.

\[\begin{align*}
E_t[X(t + T)] &= \sigma^2 X(t) + \sigma^2 u_{t+1} + \cdots + u_{t+\tau} \\
&= \sigma^2 X(t) \quad \text{since } E_t(u_{t+1}) = E_t(u_{t+\tau}) = E_t(u_{t+\tau}) = 0
\end{align*}\]

and \( X(t) \), the value of the current spot price, is already known. By the same reasoning, it can be shown that:

\[
E_{t+1}[X(t + T)] = \sigma^2 X(t) + \sigma^2 u_{t+1} = \sigma^2 X(t) + \sigma^2 u_{t+1}.
\]

Therefore, \( R(t, T) = E_{t+1}[X(t + T)] - E_t[X(t + T)] = \sigma^2 X(t) + \sigma^2 u_{t+1} - \sigma^2 X(t) \) and

\[
E[R(t, T)] = 0.
\]

If spot prices follow a second order autoregressive process \( \sigma^2(R) = 0 \) and \( \sigma^2(R) = \sigma^2(\alpha) \) as before, however, the expression for the futures return involves a second-order difference equation in spot prices. That is,

\[
R = -u(t) (\lambda_1 + \lambda_2 - \lambda_3) - \lambda_2 - \lambda_3
\]

with

\[
\sigma^2(R) = (\lambda_1^2 - \lambda_2^2) - 2\lambda_1\lambda_2 - \lambda_2^2 - \lambda_3^2 \quad \text{[\( \sigma^2(u) / (\lambda_1 - \lambda_2) \)}
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the roots of a second order polynomial. Thus, the properties of futures return volatility will depend on the numerical values of the parameters of the second-order process.
the greater the extent at which spot price (hence futures price) fluctuations will be offsetting.\textsuperscript{4}

Samuelson's explanation of the increasing volatility of a maturing futures contract is crucially dependent on the mean-reverting properties of Equation 2.\textsuperscript{5} This can be shown by examining how spot prices follow a non-mean reverting process such as random walk. The generating process for a random walk is:

\[ X_{t+1} = X_t + \{u_t\} \]

and the expression for return has the simple form:

\[ R(t, T) = u_{t+1} \]

with

\[ E[R(t, T)] = 0 \text{ and } \sigma^2(R) = \sigma^2(u). \]

Expected return is zero, but in this case variance of return is constant, independent of contract maturity, i.e., the variability effect disappears.\textsuperscript{6}

\textbf{SECTION 2: SOME EVIDENCE ON THE RELATION BETWEEN CONTRACT MATURITY AND RETURN VARIABILITY}

Daily price quotations from the June and December live-beef futures contract for the period 1965 through June, 1972, have been used to test the hypothesis that there is a systematic relation between return variability and contract maturity.\textsuperscript{7} The sample period includes 7½ sets of observations for each contract. In most cases, contracts are outstanding 10 or 11 months before maturity, although there are some cases where price quotations are available as long as 15 months before maturity. Such observations from the June and December contracts are high but not perfect — 76 for a sample size of 1,432. Hence, in the statistical tests to follow, we shall report all results by contract and not aggregate the returns.

\textsuperscript{4} Samuelson's explanation of the variability phenomenon is in no way dependent on whether futures prices exhibit normal backwardation or not. Samuelson, probably for expository reasons, assumed that futures prices follow a pure martingale. In a later section of the paper, however, he shows how the analysis can be extended to encompass a martingale with drift — a case he explicitly identifies with Keynesian normal backwardation.

\textsuperscript{5} The assumption of a stationary disturbance term also plays a crucial role in this analysis. If the disturbance term were non-stationary, no a priori pattern in return variance could be rejected. In all examples, stationarity of the disturbance terms will be assumed.

\textsuperscript{6} By the same reasoning, the same results would emerge if the first differences of spot prices followed a first-order-moving-average process. And in general, return volatility would be unrelated to contract maturity if one were to assume that any number of more complicated higher-order-moving-average processes were generating the spot-price series.

\textsuperscript{7} Trading in the live-beef futures was instituted on November 30, 1964, by the Chicago Mercantile Exchange. In addition to December and June contracts, there exist contracts which mature in April, August, October, and February. The correlation between returns on the June and December contracts is high but not perfect — .76 for a sample size of 1,432. Hence, in the statistical tests to follow, we shall report all results by contract and not aggregate the returns.
tions had to be deleted from the sample, however, since trading volume in the maturity range 12 to 15 months is typically low and the reported prices are, in many instances, only nominal quotations.

All statistical tests are presented in terms of the first difference of the daily price expressed in natural logarithms. The first difference of the daily log price is the rate-of-return continuously compounded for holding the futures contract over the day, and for small price changes. The daily log price approximates the daily percentage price change.\(^8\)

As a measure of return variability, the .28 to .72 interfractile range has been computed from daily return series for contract maturities ranging from 11 months to the expiration month of the contract. (For a description of the sampling properties of this statistic, see Fama and Roll, 1971.) An interfractile range, rather than the variance or standard deviation, was used as the primary measure of return variability since futures returns appear to conform to a member of the non-Gaussian family of distributions better than to the Normal distribution.\(^9\) Under these conditions the sample variance is known to be an inefficient measure of variability (Fama and Roll, 1971).

Estimates of the sample interfractile range are reported in Table 1 (along with the sample variance which is included for the sake of completeness). Since the interfractile range is a measure that corresponds to the standard deviation for the Normal distribution, the interfractile range has been transformed to the equivalent of the variance.\(^10\) From here on, the interfractile range will refer to this transformed value.

\(^8\) We have checked the first part of the Samuelson hypothesis that futures returns follow a martingale. Sample correlation coefficients, defined as \(\rho_\tau = \text{cov}(R_\tau, R_{\tau-\tau})/\sigma(R_\tau) \cdot \sigma(R_{\tau-\tau})\), have been computed for lags of 1 to 10 days. Evidence of serial correlation in the return series would cast doubt on the validity of the martingale assumption. There is virtually no serial dependence in the return series for either the June or December live-beef contract. Out of 20 correlation coefficients, all but four are within two standard errors of zero. The largest value of \(\rho_\tau\) is .10 (first order December) which "explains" only 1 percent of the return variability.

From these results, we may infer that the behavior of returns by maturity, obtained by reclassifying the original return series with respect to number of months to contract maturity, is also consistent with the martingale hypothesis.

\(^9\) Normal probability plots for each of the 12 maturities for the June and December contracts confirmed that the majority of the sample distribution departs from normality. If returns were distributed normally, values of the observation plotted against their fractile value would lie along a straight line having slope \(1/S\) and intercept \(\bar{x}/S\) where \(S\) is the sample standard deviation and \(\bar{x}\) is the sample mean. In only 7 of the 24 plots was there little or no evidence of curvature away from the line (7,8 month June and 10, 6, 2, 1 and maturity month December live beef).

\(^10\) For stable distributions, the second moment is defined as \(\delta = S/\sqrt{\alpha}\) where \(\alpha\) is the characteristic exponent which determines the shape of the distribution and \(S\) can be estimated by the interfractile range. Thus, to convert the interfractile range to the equivalent of the variance, \(S\) must be raised to the alpha power.
Figures in Table 1 suggest an inverse relationship between contract maturity and return volatility. For both June and December contracts, volatility appears lower in the early months (11 to 8 months for the June contract and 11 to 7 months for the December contract) than later ones and increases sharply in the maturity month.\(^\text{11}\)

The correlation coefficient between the interfractile range and length of contract maturity is .74 for the June contract and .59 for the December contract.\(^\text{12}\) If the true value \(\rho\) were zero, the figures .74 and .59 would represent normal deviates of 2.87 and 2.02. The probability of observing a sample deviate of 2.87 or 2.02 when the true value is zero is only .20 and 2.17 percent.\(^\text{13}\)

\(^{11}\) In the case of live-beef futures, the increase in the interfractile range during the contract maturity month cannot be attributed to suspension of trading limits on daily price movements. For this particular commodity the limits, which are $1 above or below the previous day's closing price, remain in effect until the expiration date of the contract.

\(^{12}\) Using the variance, rather than the interfractile range, as a measure of volatility \(\rho\) drops to .56 and .44, which is understandable in terms of the higher sampling variability of the variance, as opposed to the interfractile range, when returns are distributed stable non-Gaussian.

\(^{13}\) A second measure of association, which does not necessitate specifying the functional form of the relation between contract maturity and variability, is Kendall's tau statistic. For a description of the statistic and its sampling properties, see Kendall (1962). If there is no relationship between the number of months to con-

### Table 1

<table>
<thead>
<tr>
<th>Month to Maturity</th>
<th>June</th>
<th>December</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Interfractile Range</td>
<td>Variance</td>
</tr>
<tr>
<td>11</td>
<td>.000584</td>
<td>.000148</td>
</tr>
<tr>
<td>10</td>
<td>.000596</td>
<td>.000228</td>
</tr>
<tr>
<td>9</td>
<td>.000427</td>
<td>.000086</td>
</tr>
<tr>
<td>8</td>
<td>.000462</td>
<td>.000104</td>
</tr>
<tr>
<td>7</td>
<td>.000739</td>
<td>.000115</td>
</tr>
<tr>
<td>6</td>
<td>.001008</td>
<td>.000262</td>
</tr>
<tr>
<td>5</td>
<td>.000816</td>
<td>.000219</td>
</tr>
<tr>
<td>4</td>
<td>.000954</td>
<td>.000191</td>
</tr>
<tr>
<td>3</td>
<td>.001314</td>
<td>.000304</td>
</tr>
<tr>
<td>2</td>
<td>.000854</td>
<td>.000214</td>
</tr>
<tr>
<td>1</td>
<td>.000815</td>
<td>.000403</td>
</tr>
<tr>
<td>MAT</td>
<td>.001149</td>
<td>.000271</td>
</tr>
</tbody>
</table>

\(^a\) These variability measures, although computed from a daily price series, have been transformed to a weekly equivalent. The reason for doing this will be apparent (see Section 3: Process Generating Spot Prices).
Thus, we cannot reject the hypothesis: return variability for live-beef futures is inversely related to contract maturity.\textsuperscript{14}

\textbf{SECTION 3: PROCESS GENERATING SPOT PRICES}

If Samuelson's explanation for the existence of a volatility effect is correct, results of the previous section suggest that spot-cattle prices are generated by an autoregressive process. However, finding an appropriate set of cattle prices with which to estimate the process is not an easy task. Although the U.S. Department of Agriculture publishes large quantities of price statistics, most of the reported series are in the form of temporally averaged prices (usually monthly or weekly). Since use of average prices can "introduce correlations not present in the original series" (Working, 1960) such prices cannot be used in situations where the size of the correlation coefficients is used to establish the nature of the generating process.\textsuperscript{15,16}

Having rejected the use of temporally averaged prices, an appropriate measurement interval must be chosen. Since the sample of futures returns is based on daily prices, a series of daily spot-cattle prices is used to estimate the generating process. Unfortunately, the properties of daily spot-price changes seem to vary over the week. Using the bottom of the daily price range for Choice slaughter steers, 1,100-1,300 pounds sold at the

tract closeout and the size of the interfractile range, the value of tau is zero. The values of this statistic are .55 for the June contract and .42 for the December contract which correspond to normal deviates of 2.40 and 1.85. The probability of observing a sample deviate of 2.40 or 1.85 when the true value is zero, is only .8 percent and 3 percent. The value of Kendall's tau statistic for the June and December live-beef contracts, using the sample variance instead of the interfractile range as a measure of return variability, is .48 and .12. The corresponding probabilities of observing such values when the true value of $\tau$ is zero is 1.7 percent and 32 percent.

\textsuperscript{14}In a study of the relationship between contract maturity and price volatility for wheat futures traded on the Chicago Board of Trade, Segall (1956) concluded that his empirical evidence suggested "only the absence of an easily stated relationship between maturity and price volatility . . ." (p. 206).

From one set of test results, Segall concluded that "maturity is related to price volatility" and that some support was afforded to the position that longer maturities fluctuate less than shorter maturities. On the other hand, the size of mean and median price ranges for contracts having 1 to 8 months to maturity exhibited no regular pattern.

\textsuperscript{15}Working (1960) has shown that the use of average prices introduces spurious first-order serial correlation into the series. For instance, according to Working's formula for the first-order correlation coefficient, $(m^3 - 1)/2(2m^2 - 1)$ where $m$ is the number of terms in the averaging interval, if daily prices follow a random walk, the first-order serial correlation coefficient using a monthly average of the daily prices will be .25.

\textsuperscript{16}The fact that temporally averaged prices can introduce spurious first-order serial correlation has not deterred researchers from using such series. See for instance, the recent study of spot-cattle prices by Carvalho (1972).
Union Stockyards in Chicago from January, 1964, through July 31, 1971, the mean daily price change, its standard error, and the frequency distribution of price changes for each trading day of the week have been computed.\(^7\) (See Table 2.)

The sample frequency distributions in Table 2 show that Tuesday and Thursday price quotations are nominal ones. From Monday to Tuesday, 83 percent of the price changes are zero; the corresponding figure for Wednesday to Thursday is 98 percent. By contrast, distributions for Tuesday-to-Wednesday, Thursday-to-Friday, and Friday-to-Monday price changes are roughly similar and take on a much higher proportion of non-zero values (65, 64, and 62 percent respectively) than do Monday-to-Tuesday, and Wednesday-to-Thursday distributions.

Table 2 suggests there are at most three effective trading days in the week: Monday, Wednesday, and Friday.\(^8\) Thus, a weekly time series would be a natural choice to use for estimating the process generating spot-cattle prices. Instead, the Monday price series is used here since the largest volume of cattle arrive at the yards for sale on that day.\(^9\)

The proposition that under certain conditions return variability will be systematically related to contract maturity was demonstrated for the case of an autoregressive model that was additive in the error term. A multiplicative form of the model has been chosen to estimate the return generating process model since there is reason to believe that the size of the error term is proportional to the current price level. The relevant generating process is assumed to be:

\[
p_t = p_{t-1} \exp (\alpha + \tilde{u}_t)
\]

or taking logarithms

\[
\ln p_t = \alpha \ln p_{t-1} + \tilde{u}_t \ln p_{t-1}
\]

17 Ideally, one would want to use the closing market price for spot cattle, but since the U.S. Department of Agriculture does not collect open, high, low, or closing livestock market prices, the decision was made to use the bottom of the daily price range. The terminal date of the sample corresponds to the closing of the Union Stockyards. The weight category 1,100-1,300 pounds corresponds most closely to the delivery specifications of the live-beef futures contract. (Data were made available by John McKenna, U.S. Department of Agriculture, Chicago.)

18 A spot check of daily volume figures for the number of cattle arriving for sale at the Union Stockyards in Chicago for the years 1964, 1966, 1969, and 1970 substantiates the contention that only Monday, Wednesday, and Friday (at most) are effective trading days. Representative volume figures over the week are as follows: Monday, 11,000 head; Tuesday, 2,000; Wednesday, 9,000; Thursday, 500; and Friday, 5,000. In 1964, as many as 18,657 head of cattle arrived at the Union Stockyards on a Monday. By 1970, volume of cattle arriving at the yards had declined dramatically, e.g., Monday volume averaged in the neighborhood of 8,500 head but the Monday-Wednesday-Friday pattern was still pronounced. (Figures supplied by John McKenna, U.S. Department of Agriculture, Chicago.)

19 As noted, the results reported in the text are based on the Monday price series. All empirical tests, however, have been replicated using the Wednesday and Friday price-change series. In no case were the results substantively different from those reported in the text.
TABLE 2

FREQUENCY DISTRIBUTIONS OF DAILY PRICE CHANGES BY DAY OF THE WEEK

<table>
<thead>
<tr>
<th>Daily Price Change</th>
<th>Monday-</th>
<th>Tuesday-</th>
<th>Wednesday-</th>
<th>Thursday-</th>
<th>Friday-</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tuesday</td>
<td>Wednesday</td>
<td>Thursday</td>
<td>Friday</td>
<td>Monday</td>
</tr>
<tr>
<td>$-1.25</td>
<td>1</td>
<td>4</td>
<td>24</td>
<td>25</td>
<td>1</td>
</tr>
<tr>
<td>$-1.00</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$- .75</td>
<td>5</td>
<td>29</td>
<td>1</td>
<td>75</td>
<td>85</td>
</tr>
<tr>
<td>$- .50</td>
<td>25</td>
<td>90</td>
<td>4</td>
<td>376</td>
<td>135</td>
</tr>
<tr>
<td>$- .25</td>
<td>311</td>
<td>138</td>
<td>376</td>
<td>135</td>
<td>142</td>
</tr>
<tr>
<td>$ 0</td>
<td>85</td>
<td>81</td>
<td>81</td>
<td>5</td>
<td>1</td>
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<td>1</td>
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<td>376</td>
<td>391</td>
<td>382</td>
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<tr>
<td>$1.25</td>
<td>372</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Mean Price Change

|                    | $0.0013    | $-0.0013   | $-0.0026    | $0.0336    | $0.0040     |

Standard Error

|                    | (0.0074)   | (0.0158)   | (0.0023)    | (0.0173)   | (0.0008)    |

where $p_t$ and $p_{t-1}$ are the Monday prices of spot cattle one week apart and $u_t$ is a random error term. A random-walk process would imply that $\alpha = 1$, or alternatively that the first differences of the log cattle prices are serially uncorrelated.

Using a sample of 395 Monday spot-cattle prices, a first-order autoregressive process was estimated.

$$\ln P_t = 0.019 + 0.987 \ln p_{t-1} \quad R^2 = 0.9720$$

Since the point estimate of the regression coefficient is close to, but not actually 1, (the sampling interval of two standard errors wide around it extends from 0.971 to 1.003) the results are somewhat ambiguous to interpret. In a situation such as this, though, description of the process generating spot-cattle prices depends, in part, on what is being investigated. For instance, if the proposition under discussion is the profitability of trading rules, the conclusion is that for all practical purposes spot-cattle returns behaved as if they followed a random walk. The slight degree of dependence in the return series could not be profitably exploited once transaction costs were taken into account. On the other hand, the case may be that a value of $\alpha = 0.987$ is sufficiently different.

*A second-order price was also estimated but with no material improvement in fit, i.e., the $R^2$ increased from 0.9709 to 0.9714.*
SECTION 1: PRICE BEHAVIOR

from 1 to account for the pattern in the size of the variability measures for the live-beef futures returns. (See Table 1.)

Using the formula $\sigma^2(R) = \alpha^{2(T-1)}\sigma^2(\bar{u})$, with $\alpha = .987$ and $\sigma^2(\bar{u})$ being the interfractile range for the spot-return series, predicted values of $\sigma^2(R)$ (actually the interfractile range) have been computed for $(T - 1) = 11$ months through $(T - 1) = 0$ months and the results have been plotted in Figures 1 and 2, along with the actual interfractile ranges and the regression trend line through them. The correlation between the predicted and actual volatility measures for June live beef is .74; for the December contract it is .60.21

Figures 1 and 2 show that although the predicted interfractile range increases as the maturity date of the contract approaches, the initial predicted value at $(T-1) = 11$ months is too high, and its rate of increase

21 Assuming that the relationship between futures and spot variability is logarithmic does not materially change the results. The correlation coefficient for the June contract increases from .74 to .76 and for the December contract increases from .60 to .61. Predicted variability measures based on spot variance have also been computed. The correlation coefficients between predicted and actual variance for the June and December contracts are .48 and .19; in logarithmic form they are .45 and .17.

**FIGURE 1. JUNE VARIABILITY MEASURES**
as contract duration decreases is too gradual compared to the actual values of the interfractile ranges.

If alpha were somewhat smaller, the more distant scale factors would be lower, and the ascent to the value of the interfractile range at maturity (i.e., the actual value of the interfractile range for the spot-return series) would be steeper. Consider, for example, the pattern of the predicted interfractile ranges if $\hat{\alpha} = .97$ rather than .987. (The value .97 was chosen since it is the lower endpoint of the interval two standard errors wide around the point estimate of $\hat{\alpha}$). For $\hat{\alpha} = .97$ the predicted values of the interfractile range closely follow the trend line of the actual interfractile ranges. For the June live-beef contract, 9 out of the 12 predicted values of the interfractile range lie almost exactly on the trend line. Correspondence between the predicted and actual variability measures is not as close for the December contract, primarily because the value of the interfractile range at maturity is more than twice the size of the next largest interfractile range. Were this last value to be somewhat smaller, the upward trend in the size of the December variability factors would be less pronounced and would follow more closely the plot of the predicted variability measures for $\hat{\alpha} = .97$. 

FIGURE 2. DECEMBER VARIABILITY MEASURES
This last example was chosen to demonstrate that at least one value of alpha exists for which the correspondence between the actual and predicted variability measures would be very close. However, given a sample value of $\alpha = .987$ and a standard error of $.008$, the probability that the true value of alpha could be as low or lower than $.971$ is only $.0166$.

Insofar as returns on the live-beef futures are concerned, this evidence on the relation between return volatility and contract maturity should be interpreted rather cautiously. There does appear to be a systematic variability effect present in the structure of futures returns; whether this effect is attributable to the process generating spot prices is open to question. There is certainly a rough correspondence between the predicted and the actual variability measures based on a value of $\alpha = .987$. The fit between predicted and actual interfractile ranges is even closer when $\alpha = .97$, but as we have seen, such a value would probably be rejected as the true value of alpha.

Thus, although return variability on the live-beef futures contract is related to contract duration, the explanation for the phenomenon is probably due only in part to the process generating the spot-cattle price series. Whether these results are characteristic of all commodities, or only live beef, can be answered only by examining more commodities.

REFERENCES


