The Supply of Storage for Frozen Pork Bellies
John C. Pickett

A number of recent authors (Brennan, 1958; Samuelson, 1966; Stein, 1961; Telser, 1958; Tomek and Gray, 1970; and Weymar, 1968) have examined temporal price relations for various agricultural commodities. This study is an application of the theory developed by these authors to frozen pork bellies which are traded on the Chicago Mercantile Exchange.

Temporal price relations are concerned with the structure of prices for successive time periods. These prices may be the current spot price and the past and future spot prices. Also, these prices may be represented by the current spot price and the corresponding prices for futures contracts. In the case of frozen pork bellies, the futures contracts which are currently traded are February, March, May, July, and August. On any given trading day, a spot price exists along with a price for each futures contract.¹ The relationship of these spot and futures prices over time is the essence of temporal price analysis.

Futures markets perform two important functions; they act as a guide for inventory levels, and establish forward prices. Both functions are so closely intertwined that evidence of their separate performance has not been stressed (Tomek and Gray, 1970). This paper examines the first function and assumes that the participants determine the second by their actions in the market as they react to existing and anticipated inventory levels.

As a consequence of the importance of inventories on futures prices, futures markets have originated and have had their highest development occur in situations where stocks of annual crops are held continuously (Tomek and Gray, 1970). Frozen pork-belly stocks are held continuously, in cyclical quantities, but are not a typical annual crop. Two crops are generally produced each year resulting from the biologically imposed breeding-maturing requirements. Since there are two crops each year and

¹ If the old contract has matured and the new contract has not begun to trade, then the one or more contracts may not be quoted on a particular day.
consumption occurs throughout the year, a major role of the pork-belly futures market is the temporal allocation of the seasonal inventories throughout the consuming year. The accumulation and diminution of commercial inventories are guided by the relationship between the cash and futures prices. As these inventory adjustments are frequently accompanied by the sale and purchase of futures contracts, the price relationships between the spot and various futures contracts closely reflect the inventory positions.

THEORETICAL MODELS

The Price-Spread Model

Samuelson (1966) has applied the tools of spatial competitive relations to equilibrium commodity prices over time. Consider an agriculture commodity whose harvest occurs one day during the year but whose consumption is spread evenly throughout the year. The essential question is: What set of prices will have to exist in order to allocate the one-time harvest of the crop among the consumers who demand the commodity continuously throughout the year? The intuitive answer is obvious. The price will have to rise continuously from one harvest to the next, inducing an individual to store a portion of the crop until it is required by the consumers. Figure 1 shows the crop available and successive harvest periods. Figure 2 is the inverse of Figure 1, and shows the movement of prices for successive harvest periods. Figure 3 shows the supply-and-demand curves which can generate the price path traced in Figure 2. Figure 3 is a typical two-market, back-to-back diagram. The one-time harvest supply is $S_1$ and the demand throughout the year is represented by $D_1$ and $D_2$. Market 1 represents the harvest market or time period, while Market 2 represents the
post-harvest market or time period. \( T_{12} \) is the cost of storing the commodity from Time Period 1 to Time Period 2.

Carryover of the commodity from Market 1 to Market 2 will occur only if \( A_2 \geq A_1 + T_{12} \). (Note: the horizontal axis in Figure 3 for Market 2 is below the horizontal axis for Market 1 by cost of storage, \( T_{12} \).) Equilibrium between the two markets (time periods) will occur at the price indicated in Figure 3, where \( A_2 = A_1 + T_{12} \). If the price in Market 2 does not exceed the price in Market 1 plus storage costs, no carryover will occur. If the price in Market 2 does exceed the price in Market 1 by more than storage costs, storage will be stimulated, lowering the price in Market 2 and raising the price in Market 1 until equilibrium is reached. This distributes the harvest from one time period to the next.

Later time periods can be depicted by similar diagrams with \( T_{34} > T_{23} > T_{12} \) which represents higher storage costs for longer storage periods (additional days, weeks or months). The \( A_2 \) price in Figure 3 is just one point on the curve in Figure 2. Each point on Figure 2 can be generated by a Figure 3 with the higher storage costs representing longer storage periods. Hence, imagining a whole series of diagrams like Figure 3 in succession from harvest-to-harvest will generate the pattern of prices traced in Figure 2. Price changes within the crop year are related to storage costs. Prices drop in the next harvest period when the new crop is available, and the price level from crop year to crop year can vary according to demand-and-supply conditions each year.

Figure 4 is constructed from Figure 3 in order to determine the amount of carryover. Curve \( ES_1 \) is the excess supply curve of Market 1 and is generated by subtracting \( D_1 \) from \( S_1 \) (Figure 3) at all prices. \( ES_1 \) will be zero where \( D_1 = S_1 \), the market clearing price in Market 1. Curve \( ES_2 \) is the excess supply curve of Market 2 and is generated from \( D_2 \) and \( S_2 \).
similar to the process for $ES_1$. Since Market 2 represents a post-harvest time period, $S_2 = 0$, which results in $ES_2$ being the negative of $D_2$. The point where $ES_1$ and $ES_2$ intersect in Figure 4 represents the equilibrium price between Market 1 and Market 2 and the quantity of carryover stocks between the two time periods if there are no storage costs, or $T_{12} = 0$.

Since storage costs are involved, Figure 5 is constructed from Figure 4. The vertical axis measures the price spread, or storage costs, from Market 1 to Market 2. Curve $NN$ is constructed by subtracting $ES_1$ from $ES_2$ at each quantity point, the vertical distance between the two curves. Curve $OAT$ measures the constant storage costs required to transfer the commodity from Time Period 1 (Market 1) to Time Period 2 (Market 2). Distance $OA$ equals $T_{12}$. The amount of stocks which will be carried over is determined by the quantity $ES_2 - ES_1 = T_{12}$. The equality holds at point $K$ and the amount of carryover is $OL$.

Figures 3 through 5 can be used to trade the effects of a change in supply and/or demand in either market on the size of the carryover stocks. For example, if supply increases in Market 1, then $ES_1$ shifts to the right, $NN$ shifts up, resulting in larger stocks being carried over given constant storage costs.

A refinement of the storage cost curve, $OAT$, would allow a maximum amount of carryover stocks. This maximum would be determined by the existing storage capacity. Additional quantities could not be stored at any price which is depicted by the vertical portion $TU$.

Actual storage costs can be allowed to vary as the size of the carryover stocks vary. This is a more attractive assumption, since at low carryover
quantities warehouse operators would be willing to lower their prices as the demand for storage space declines. At high levels of carryover stocks, additional storage space could be obtained by using the more expensive storage facilities and converting storage space, normally used for another commodity, to the high storage price commodity. Figure 6 demonstrates the original fixed-price storage curve, OATU, and the variable-price storage curve, BC. Working, in a classic analysis, conceived this functional relationship as a storage supply curve (Tomek and Gray, 1970).

The objective of the supply-of-storage theory is to explain intertemporal price differences between spot and forward prices or between spot and expected future spot prices (Weymar, 1968). The cash price depends on the supply-and-demand conditions at any moment in time, and the supply-and-demand conditions in adjacent time periods. A future price for a
particular delivery month depends on the supply-and-demand conditions expected to prevail between the current period and the delivery month. Thus, the level of prices for all delivery months, including the nearest month and the cash price, respond to changes in information concerning the relevant supply-and-demand conditions. The interrelationships between successive prices may also change, but mainly in response to the level of current inventory (Tomek and Gray, 1970).

As demonstrated in Figure 6, the spread between the future price and the spot price is a function of inventory levels. This spread is the price of storage and is normally positive. A positive storage price reflects the carrying charges of inventory from one time period to the next. An inverse carrying charge is said to exist when this price spread is negative. An inverse carrying charge indicates that a scarcity exists for current inventory, causing the cash price to exceed the future price plus storage costs. In Figure 6, a negative price spread occurs at low inventory levels.

The Holding Cost Model

Consider a commodity that has no organized futures market such as inventories for manufactured goods. What benefits do these inventories provide to their owners? There are two types of yields that may accrue from holding such inventories:

THE STOCKOUT YIELD

Inventories eliminate the possibility that an order cannot be filled because the production run cannot produce the amount that is ordered within the designated delivery period.
THE COVERAGE YIELD

A second yield is produced by reducing the frequency of price changes for the finished goods. Stability of finished-goods prices is encouraged by altering the price only when raw-material prices deviate significantly from the prices of finished goods. An inventory of raw materials requires that a large quantity of that inventory be put into the production process before the replacing inventory prices cause the finished-goods prices to be altered. This yield is nothing more than stating that a large inventory makes average raw-material prices change more slowly than a small or nonexistent inventory which reduces both frequency and size-of-price changes for the finished goods.

The marginal inventory holding cost \( (MC_s) \) is equal to the storage costs \( (C_s) \), minus the stockout yield \( (R_s) \), minus the coverage yield \( (R_c) \) (Weymar, 1970).

\[
MC_s = C_s - R_s - R_c
\]

(1)

Storage costs include the normal warehousing, insurance, and interest costs of holding a unit of inventory. Figure 7 shows the relation of storage costs and inventory levels while Figures 8 and 9 show the relation of stockout yield and coverage yield and inventory levels. Note that in Figure 9 coverage yield becomes negative for large inventory positions, since price rigidity in the face of competitor price reductions produces a negative yield for the large inventory. If \( MC_s \) is calculated at each price by Equation 1, Figure 10 will result. Note that at large inventory levels, \( MC_s = C_s \) since \( R_s \) approaches zero and \( R_c \) becomes zero then negative for larger inventories.

The functions in Figures 6 and 10 are equivalent. The discussion preceding each figure develops the function in a different manner, but the results are the same. The equivalence of both arguments is important here. The former back-to-back, supply-and-demand curves are used to generate excess supply curves which show the amount of inventories carried from one time period to the next. In the latter discussion, the marginal storage cost has three separate components each of which varies as the inventory level varies. The marginal storage-cost function is obtained by the linear combination of the three components at each inventory level.

Implicit in the analysis has been the economic motivation of the individual who holds stored inventory as either the price spread varies or as the inventory yield varies. The economic function performed by the holder of seasonally produced commodities is to provide storage services to the market. He is being paid the storage costs \( T_{12}, T_{23}, \) and so on. The holder of commodity inventories for which no futures market exists requires such inventories for manufacturing activities. His return is reflected in the
profits of his manufacturing activities. Storers and processors are not the only holders of inventory. Speculators and dealers, acting in a speculative capacity, will hold inventory if they expect the price to appreciate at a rate fast enough to cover carrying costs and yield a satisfactory return (Weymar, 1968). The total inventory is then held by processors and speculators, given the price spread, or the expected rate of price appreciation.

Either of the two theoretical formulations can be used as the model when an empirical estimation of a supply-of-storage function is undertaken. One of the formulations must be chosen before estimation can be undertaken and the supply-of-storage function actually estimated.
STATISTICAL ANALYSIS

Assumptions

A fundamental problem—what assumptions are necessary before the storage function can be estimated—must be resolved before empirical estimates can be discussed. Assuming that the storage function is relatively stable, and the demand curve varies, a supply function can be generated by plotting the price spread against inventories. Assuming that neither the supply nor the demand curve is stable, the plot-of-price spreads against inventories produces a series of discontinuous points unrelated to each other, because different supply-and-demand curves generate each observed point. If different supply-and-demand curves hold for each point, then it is impossible to estimate one supply-of-storage function, unless a system of simultaneous equations is constructed. If the demand curve is stable and the supply curve varies, then the plot-of-price spreads against inventory generates a demand for storage function. This paper assumes that the supply-of-storage function is stable and the demand curve varies along this supply function. Empirical research indicates that the components of the cost of storage have been relatively stable over time; hence the supply of storage is stable (Brennan, 1958).

Estimating procedure requires specification of one of the two theoretical models. The availability of data has forced the use of the price-spread model here. The vertical axis of Figure 10 is identified as costs, while the price-spread model identifies this axis as the spread between the future and the cash price. There is difficulty in obtaining not only estimates of the storage costs and stockout and coverage yields, but also in the functional relationship between quantities and inventory positions. Absence of the components of the marginal holding-cost function leads to the necessity that the price-spread model is the theoretical model being estimated. Price spreads are readily obtainable from published sources.

Data

Two types of data are needed to estimate the supply-of-storage function. The first is price data necessary to construct the price spreads. These data are obtainable from the Chicago Mercantile Exchange Annual Yearbook which contains daily price quotations for each futures contract and the cash price for different weight frozen bellies. Data points used in estimating procedures are the weekly Friday closing price for each futures contract and the weekly Friday closing price for 12-14 pound cash frozen bellies recorded in dollars and cents. Originally, the contract specifications called for delivery of 12-14 pound bellies but currently, delivery can be...
made in 10-14 pound bellies. The cash price quotation used is frozen 12-14 pound bellies. Periodically, the market is closed on Friday. The weekly closing price for both the futures contract and cash is then the last trading day of the week.

The second data requirement is inventory positions. These data are contained in both the *Annual Yearbook* and *Cold Storage Report*. The *Annual Yearbook* contains inventory positions in two geographical locations — warehouses are located inside Chicago or outside Chicago. Total holdings in each location are available on an end-of-week basis in units of 1,000 pounds, and are used as the data for inventory positions. The number of warehousing facilities contained in the sample varies during the time period 1965 to 1970. When a change in the number of warehousing facilities occurs, no significant change can be detected in the inventory positions. Therefore, no adjustment is made on the inventory positions for the number of facilities included in the sample.

End-of-month figures for frozen pork-belly stocks are published by the U.S. Department of Agriculture in *Cold Storage Report*. The inventory positions contained therein are representative of total frozen-belly holdings in the U.S. These positions are not used in the analysis for two reasons: 1) the number of data points is reduced by a factor of four when compared to the weekly positions available in the *Annual Yearbook*, resulting in a loss in degrees of freedom in the estimated equations; and 2) some portion of total holdings of frozen bellies reported in *Cold Storage Report* are not deliverable against the futures contracts traded on the Chicago Mercantile Exchange because they are not held in approved warehouses as are the warehouses identified in the *Annual Yearbook* totals.

In addition, inventory positions do not specify the amount held in each weight class. Contracts specify delivery of 10-14 pound bellies that should be the inventory positions which are related to the price spread. Other weight classes are acceptable for delivery, but a price concession is made for weight groups other than that specified under the contract-delivery conditions. Since storage positions for 10-14 pound bellies could not be separated from storage positions of all weight classes, there was no recourse from using the storage of all weight classes.

The economic magnitude reflected in the inventory positions conforms to the variable measured along the horizontal axis in the price-spread model — carryover stocks. The observed inventory positions are: the beginning inventory, plus production or flow into stocks during the period, minus consumption or withdrawals from stocks during the period. The quantity of stocks is the variable that measures the amount to be carried from one time period to a later time period.
The time period for the analysis is from the week ending June 5, 1964, to May 15, 1970. The starting date marks the beginning of inventory positions for both inside and outside Chicago in the Annual Yearbook. The ending date is the last date of the maturing May, 1970 futures contract.

The interval between data points is one week. An interval shorter than one week would not have inventory positions available from the Annual Yearbook. An interval of one day would serve two functions: 1) to introduce inconsistencies in the data interval resulting from holidays and weekends, and 2) to magnify the effects of not only speculative inventory positions but also the effects of speculative activity in the price of futures contracts. Fortnightly or monthly data intervals were not used because of the loss in degrees of freedom.

An understanding of the methodology followed in constructing the price spreads depends on the institutional arrangement for quoting prices for future contracts. At any given point in time, a price is quoted for February, March, May, July, and August maturing contracts. The cash price could be subtracted from each quoted price to construct a spread between a fixed date in the future for which a known price exists and the cash price. For any given week, a price spread is constructed between the Friday closing price for each futures contract and the Friday cash price. In moving through time, a set of price spreads is constructed for each contract for each week. As the old contract matures and the new contract begins to trade, the new futures contract price is substituted for the old. This procedure results in a set of price-spread observations for each contract from 1964 to 1970. The set of price-spread observations does contain missing observations for those weeks where the old contract has matured but the new contract has not yet begun to trade.

The nonavailability of data in appropriate form presents an additional problem. The price-spread model related the amount of carryover stocks to a price spread between only two time periods. The inventory figures used in the analysis can be related to a maximum of five different price spreads. For example, visualize a table that includes time for rows, and the price spread for each of five futures contracts, and the inventory position representing columns. This inventory position can then be related to each of the price spreads. An inventory position for a January date can be used to deliver against any contract during the year. To conform precisely to the model, the inventory positions held to settle each of the maturing contracts would need to be available. However, data are not available in this form so the total inventory position must be related to the price spread of each contract maturity.
Statistical Estimates

Construction of the price spread for each contract maturity throughout the time period is a straightforward procedure when utilizing Equation 2:

\[ P = P_f - P_c \]  

(2)

where \( P_f \) is the futures price and \( P_c \) is the cash price. Discussion of Figure 3 indicated that, as the time period between harvest and consumption lengthens, the storage costs of transferring the commodity between time periods increases. As the price spread is now expressed in Equation 2, no adjustment is made to differentiate a contract that has only one month until maturity and one that has 11 months until maturity. This absolute price difference will automatically change, not only as inventory positions are altered, but also as the maturity date approaches the current date. In order to eliminate the effect on the price spread of the time to maturity, price spread is adjusted to reflect the average price spread per week to maturity. This adjustment is accomplished by Equation 3:

\[ P = (P_f - P_c)/(t + 1) \]  

(3)

where \( t \) = number of weeks until the contract matures.

One is added to the denominator to eliminate the infinite value for the price spread in the last week before contract maturity and to allow for the possibility that the last trading day will occur sometime during the following week but not on the Friday of the following week.

An investigator has no knowledge a priori whether or not the average price spread per week is a correct adjustment procedure for statistical purposes. Each of the estimated equations which follow were reestimated using the unadjusted price spread of Equation 2 instead of the adjusted price spread of Equation 3. In every case the results were better using Equation 3, as evidenced by a \( R^2 \) approximately double that of \( R^2 \) using Equation 2 and higher F-values in all cases when Equation 3 was used. All following discussions of price spreads are in reference to the spread calculated by Equation 3.

A supply function always has an implicit time dimension associated with it. By a time dimension, one is implying that alternative quantities will be placed on the market at different prices per same time period such as quantities placed on the market during a week, month, quarter, or annually. The supply of storage for frozen pork bellies has a time dimension of one week, and more than one supply-of-storage function exists during a calendar year. These multiple storage functions are a result of the number of observations which exist for the futures prices. The precise definition of a storage function requires that a price spread be constructed.
between the cash price and all futures prices. A set of all futures prices does not exist. Only futures prices which are represented by the traded futures contracts exist at any moment. Therefore, a price spread can be constructed between the known cash price and the known (traded) futures contracts. At any given point in time, a price spread can only be constructed for those futures prices represented by the traded futures contracts—February, March, May, July, and August.

The fixed number of known quoted futures prices allows for the generation of a number of supply-of-storage functions revealing shifts in the function within a calendar year and between years. For example, a storage function can be estimated from the first complete set of data using the March, 1965 futures contract price. A second storage function can be estimated using the May, 1965 futures contract price. Additional storage functions can also be estimated using the July, and August, 1965, February, 1966 contract prices and continuing until May, 1970. Note that not one but a series of storage functions are being estimated.

**Methodology**

The price-spread model, which provides the theoretical foundation for the empirical investigation, fundamentally asks the question: What determines the quantity of a commodity that is carried over from one time period to a later time period? The model defines the determining factors as the price spread between the two time periods, and the cost of storing the commodity. The model can be expressed in functional form as:

\[ Q = f(PC) \]  

(4)

where

- \( Q \) = carryover stocks or inventory
- \( P \) = price spread as defined in Equation 3
- \( C \) = storage costs.

Equation 4 symbolically shows that carryover stocks depend on the price spreads. Omitting \( C \) from this equation results in the uncomplicated notation of the supply-of-storage function.

Differentiation must be made between this supply-of-storage function and a storage function expressed as Equation 5:

\[ P = f(Q). \]  

(5)

Either equation may be the correct expression of the direction of dependence between price and quantity. No general rule or set of criteria is available to choose between the two expressions. The choice depends on the model underlying the analysis. The price-spread model states that the amount of inventories carried over from the current time period to a
future time period depends on the difference in prices between the two
time periods.

The methodology for the statistical analysis is as follows: 1) indicate
the plots of stored inventory and price spreads; 2) demonstrate the results
of the simple linear-equation model; 3) show why a model which includes
lagged variables may improve the estimate of the storage function; 4) dem­
onstrate the results of the linear model which includes lagged variables;
and 5) outline some statistical problems contained in the results.

PLOTS OF RAW DATA
The first step in analyzing the data is to plot the quantities of stored bellies
against the price spread which existed for each maturity date. Figure 11
is the pictorial representation of the data. At first glance there does not
seem to be any relation between inventories and price spreads. An obvious
relation would be indicated when the pattern of points seems to trace a
straight or some type of curved line. The pattern seems to be best repre­
sented by a horizontal line parallel to the quantity axis. This is similar to
line \( AT \) in the theoretical model as drawn in Figure 6.

Remember that Figure 11 contains all the data points generated during
the sample time period. If the storage function has been slowly shifting
during the sample period, the plot of the raw data would not be expected
to reveal a perfect supply-of-storage function. The plot of raw data might
suggest that the intersection of the cost-of-storage curve and the supply­
of-storage function is being observed. Actually, a set of equilibrium points
is observed because both the cost-of-storage and the supply-of-storage
function jointly determine the inventory carried from a current time
time period to a future time period.
SIMPLE LINEAR EQUATION
Since the plot of all data points is best represented by a straight line parallel to the inventory axis (labelled $Q$) an equation of the form $Q = \alpha + \beta P + \epsilon$ was estimated. The estimated equation over all the data is:

$$Q = 34134 + 9503P \quad R^2 = .02$$

(637) (1734) $F = 15.3$

Durbin-Watson = .03.

The standard error of each coefficient is shown in parentheses beneath the coefficient.

As indicated in the preceding discussion, there are 27 subperiods in the data. Each represents the price spread for a mixed maturity futures contract. There are five futures contracts traded during the time period of the data—February, March, May, July, and August. Equations of the form of Equation 6 were estimated for each subperiod. Presentation of only the $R^2$ values for each subperiod is sufficient to point out that the estimated equations explain less than 18 percent of the variation in the quantity of stored inventory in any one case, excluding the May contracts. Table 1 presents the $R^2$ value by year and futures contract maturity. The extremely low $R^2$ values alone are grounds for rejecting the underlying price-spread model as the theoretical foundation for the analysis. The four $R^2$ values greater than 50 percent for the May contract are significant to the analysis for two reasons: 1) the price-spread model does explain more than one-half of the total variation for the four May contracts, and 2) the four May contracts produce an $R^2$ considerably higher than the other contract months for four consecutive years which may indicate that the price-spread process determining stored inventory positions differs for future contract-delivery months.

INCLUSION OF LAGGED VARIABLES
The values in Table 1 may indicate either that the theoretical model is incorrect, or that independent variables have been omitted which would help to explain the variation in the dependent variables. Given all the other professional articles which employ the price-spread model and the strength of Samuelson’s original work, rejection of the price-spread model as the basis for this investigation seemed ill-advised. (Variables are assumed to have been omitted from the equation.)

The next question to examine is to identify the variables (factors) that would explain additional variation in the dependent variable. A number will immediately come to mind, such as the price of competing meat products, supplies of competing meat products placed on the market,
income levels of consumers, and consumption of pork bellies as measured by bacon slice. These and other variables are not included for two reasons:

1. A desire to avoid the identification problem caused by including some variables that reflect both the supply-and-demand relationships. For example, if a variable for consumption (bacon slice) is included with the price spread, the estimation will reflect combined effects of supply and demand which cannot be separated. The statistical results would be difficult to interpret.

2. The relation between these omitted variables and the included variables. All other variables act to position the supply-and-demand curves in the two time periods that interact to produce two prices. This paper is not interested in defining the position of the supply-and-demand curves for the commodity in each time period, but does seek to clarify the amount of the commodity that is transferred between time periods. All the effects of the omitted variables are reflected in the two prices, and if the variables are included, the identification problem will result.

There are two types of omitted variables — lagged-price spreads, and lagged quantity of stored inventory. Justification for including lagged-price spreads depends on the decision process followed by those individuals who store bellies. The process is indicated to be that past prices are considered by individuals supplying storage before storage of bellies is provided. The market evaluates or weighs the set of current and past price spreads before storage is provided.

The lagged quantity of stored inventory is included on the grounds that there is a distinct time lag in the ability of the market to accumulate or
dispose of bellies. Accumulation of bellies is dependent upon hog slaughter and consumption, which in the short run responds to hog prices and bacon prices, and in the long run responds to the breeding-slaughtering cycle which itself is dependent upon hog prices. The disposal of bellies requires some time period, because an increase in quantities has to act to decrease bacon prices which increases consumption. If the market in the aggregate attempts to dispose of bellies in large quantities, then the price will decline, inducing some sellers to alter their decisions and hold or become buyers. The inability of the aggregate market to accumulate or dispose of belly stocks in a short period is reflected in inclusion of the lagged-inventory position as an independent variable.

The next question to answer is: How many lagged-price spreads to include as independent variables? Market participants generally consider all past prices in their decision process when the recent past is weighted heavier than the distant past. All past prices cannot possibly be included as independent variables, since there is a maximum of only 52 observations on each subperiod.

The procedure used to determine the maximum number of past prices was spectral analysis. Spectral analysis is a complicated procedure to convert the time domain to a frequency domain. One aspect of spectral analysis is a coherence function that measures the correlation between two variables in the frequency domain. Conversion from the frequency domain back to the time domain allows estimation of the relative importance of each of the past prices.

Examination of the coherence function produced a maximum of six immediate past prices which were relevant in explaining the variation in stored inventory. Since the data interval is one week, the current price spread plus the preceding six weekly price spreads, are the price spreads included as independent variables.

The lagged-inventory positions are limited to the one for the immediate past. This follows from the Koyck lag technique on a distributed lagged equation. Inclusion of lagged-dependent variables as independent variables in excess of the one in the immediate past period would present the estimating equation in an autoregressive form. In autoregressive form, current values of a variable can be estimated for all the past values of that variable. An autoregressive scheme is not assumed by the price-spread model, but could be formulated into the price-spread model.

Inclusion of lagged prices and inventory can be expressed in functional form as Equation 7. The subscripts note the relevant time period associated with each variable.

\[ Q_t = f(P_t, P_{t-1}, P_{t-2}, P_{t-3}, P_{t-4}, P_{t-5}, P_{t-6}, Q_{t-1}). \] (7)
RESULTS OF THE INCLUSION OF LAGGED VARIABLES

The estimating equation is in the form of Equation 8. This is a linear equation which is a consequence of attempting various nonlinear equations to find the best fit. None of the nonlinear forms produced fit consistently higher than the linear equation:

\[ Q_t = \alpha + \beta_1 P_t + \beta_2 P_{t-1} + \beta_3 P_{t-2} + \beta_4 P_{t-3} + \beta_5 P_{t-4} + \]
\[ + \beta_6 P_{t-5} + \beta_7 P_{t-6} + \beta_8 Q_{t-1} + \epsilon_t. \]  (8)

The desire is for all hypothesized variables defined in Equation 7 to be supported by statistical analysis. Initial estimates of Equation 8 for all sub-periods were initiated. Examination of the t-values associated with each regression coefficient indicated that in most cases some of the independent variables were not significant. A reestimation was made including only those variables that were significant at the 10-percent level. The estimated equations are shown in Table 2, identified by the sample subperiods. Standard errors have been omitted.

Interpretation of Results

Consider the first equation for March, 1965. Of the seven possible prices and the lagged-dependent variable, only the price spread lagged four weeks is statistically significant. The sign of the coefficient is negative, yet the price-spread model for the supply function hypothesizes a positive relation between price spreads and quantities. The sign of the constant or intercept is positive and the model hypothesizes a positive sign. The \( R^2 \) is extremely low and the F-value indicates the equation should be rejected, which means that either the model or the functional form (linear) of the equation should be rejected. A relevant observation is the insignificance of the lagged-dependent variable. The statistical omission of this variable suggests that the one-week lagged quantity does not enter the market’s decision-making process.

The second equation is a better estimate for discussion. All prices and the lagged quantity (inventory) are statistically significant. The correct interpretation of the regression coefficients is: once the equilibrium that exists between price spread and inventory is disturbed, the effect of a one-unit change in price in the current period on the inventory is to reduce inventory by 3,273 units if the price change is positive, and by -3,273 units if the price change is negative. The adjustment back to equilibrium does not cease, since the effect on inventory is 26,980 units resulting from the price-lagged one period, 47,164 units from the price-lagged two periods, and so on until the effect on inventory from the price-lagged six periods is 29,591 units. The total effect on inventory is the sum of all the

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<table>
<thead>
<tr>
<th>Time Period</th>
<th>Constant</th>
<th>$P_1$</th>
<th>$P_{t-1}$</th>
<th>$P_{t-2}$</th>
<th>$P_{t-3}$</th>
<th>$P_{t-4}$</th>
<th>$P_{t-5}$</th>
<th>$Q_{t-1}$</th>
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* Equation is statistically significant.
regression coefficients. In this equation the total effect on inventory is 213,045 units distributed among the lagged prices as indicated by the regression coefficients.

The effect on inventory does not cease after the sixth-lagged price. The lagged inventory is significant. The coefficient is $1 - \lambda = .89$, therefore $\lambda = .11$. The effect on current inventory of inventory-lagged one period is .89 of the preceding inventory figure. A large $1 - \lambda$ value indicates that inventory quantities are adjusting slowly. A small value for $1 - \lambda$ signifies inventories are adjusting rapidly. The Koyck lag technique demonstrates that the effect of lagged inventory on current period inventories at the cessation of the free lag is dependent upon the magnitude of $1 - \lambda$.

A pictorial representation of the lag structure for the May, 1965 equation is shown in Figure 12. The graph line connecting the points portrays the effect of lagged prices on inventory. The tail of the graph depicts the effect of the lagged inventory on current inventory. A value of $1 - \lambda$ close to one makes the line approach the horizontal axis slowly, indicating that the process generating the movement back to equilibrium is lengthy. A small $1 - \lambda$ indicates that the movement back to equilibrium occurs quickly. Of considerable importance to the analysis, is the observation of the magnitude of $1 - \lambda$ for each equation. A preceding section of this paper noted that both the original price and inventory data contain the combined effects of hedging and speculative decision making. The largest speculative activity in bellies occurs in the February, July, and August futures contracts. The estimated $1 - \lambda$ values of these three months are
typically lower than the values for the March and May contracts. Since it is impossible to disentangle the hedging and speculative activity in the data, and knowing which trading months usually receive the most speculative interest, the $1 - \lambda$ values indicate that the movement back to equilibrium is faster in those months when speculative activity is concentrated. In the two months that do not receive much speculative attention, especially May, the adjustment is much slower. The important conclusion is that the time required to restore equilibrium is short when speculative activity is large.

Returning to the discussion of Figure 12, the combined effect of all prices on the stored inventory can be represented by the sum of the regression coefficient associated with each price variable. This sum is represented by the area beneath the line connecting each regression coefficient. In estimating a supply function, any increase in price should have a combined effect of increasing the quantity supplied. Table 3 presents the sum of the regression coefficients for each subperiod. The entries for the May equations are those which result from a reestimation of these subperiods as presented in the following subsection, Statistical Problems.

An inverse relationship is indicated between price and quantity in 9 of the 27 subperiods. An increase in the price spread produces a reduced supply of stored inventory. Two possible explanations are available to explain these temporarily perverse storage functions: 1) speculative activity may completely dominate the hedging activity, or 2) the current period's supply of stored inventory which has been committed for sale exists in such a small quantity that a large price spread will not induce carryover because uncommitted stocks do not physically exist. Neither explanation can be proven either true or false. These equations which produce an inverse relationship between stored quantity and price must not be considered as the correct functional relationship between quantity and price.

The equation for February, 1969 does not contain a significant lagged-dependent variable. A hypothesized explanation is that speculative activity

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does not consider the past pattern of stored inventory as an important variable when formulating future price expectation. Therefore, the lagged-dependent variable will not be statistically related to the dependent variable. Since the lagged-dependent variable has been found to be significant in the other estimated equations, this equation is probably not representative of the actual storage function for this period. The March, 1965 equation also does not contain the lagged-dependent variable.

The equation for March, 1969 has a negative sign on the lagged-dependent variable. This sign should be positive. A negative sign is indicative of an explosive model; \(1 - \lambda = -0.11\) produces \(\lambda = 1.11\). Current inventory positions will be an increasing function of past inventory figures which is not an acceptable result given the structure of this market. The model should be rejected as an explanation for the supply of storage during this subperiod. In examining each of the remaining equations, three general observations can be made. First, there is no consistent pattern to the statistically significant variables which explain the quantity of inventory. There is no evolutionary pattern through time, and there is no pattern for a given future month between successive years. One would hypothesize a priori that the prices should enter into the decision process in an indeterminate pattern. Decision makers would observe a pattern and, by engaging in speculative activity, eliminate it. As a result, one observes that the statistically significant prices vary among the subperiods in a random pattern.

Second, in these equations where all variables are significant (especially May) there is no pattern as to the size of the regression coefficients. The contribution one lagged price makes in explaining the inventory varies among the subperiods in a random manner.

Third, the constant term is positive except in only three subperiods, indicating that the supply function is positioned as the theoretical model hypothesizes.

STATISTICAL PROBLEMS

Examination of the F-statistic indicates that the March, July, and August, 1965, August, 1968, and February and March of 1969 equations should be rejected. Those equations which are accepted are indicated by an asterisk on the F-value. Rejection of these equations implies that the equation, and therefore the model, is incorrect for these subperiods. Three of the six rejected equations are also rejected for not possessing a lagged-dependent variable or having a lagged-dependent variable of incorrect sign.

A serious problem arises with the evidence of serial correlation in all of the May equations. One assumption necessary for the ordinary least-
squares estimating technique is that the disturbances in one period are independent of disturbances in any other period. Evidence of serial correlation means that the disturbances in one period are dependent upon disturbances in earlier periods. The result of serial correlation is that the $\alpha$'s and $\beta$'s are efficient but the sampling variance is incorrect. The consequence of an error in the estimate of the sampling variance occurs when one is attempting to predict. The prediction will be biased.

Why has serial correlation arisen? There are two possibilities: 1) exogenous variables have been omitted from the equation, or 2) the functional form of the equation has been misspecified. With respect to the first possibility, the price-spread model has expanded to include lagged variables but not other variables which may generate an identification problem. For the second possibility, no other functional form has eliminated the presence of serial correlation.

No reason can be found to explain why the presence of serial correlation occurs only in the May equations. If variables have been omitted, serial correlation would be expected to occur in the other contract maturity dates. If the functional form is incorrect, the incorrect specification would be expected to appear in the other contract maturity dates as well.

A statistical procedure exists — the grid search method — for removing the serial correlation, although specification of either the omitted variables or functional form as the cause would be preferred. All significant variables have been adjusted initially by Equation 9 with $\rho$ taking the values +.99, +.98, ..., +.02, +.01.

$$X_t = \rho X_{t-1}$$

The equations have been reestimated for each value of $\rho$. Choice among all the estimated equations is made on the basis of minimum standard error of the residuals. The $R^2$ value is recorded, but is meaningless when this adjustment in the original variables is made. The Durbin-Watson statistic has improved considerably. These equations seen in Table 4 should be substituted for the May equations presented in Table 2.

**SUMMARY**

In order to summarize the results of this study, characteristics of the estimated equations must be interpreted in light of the deficiencies of the data used in the analysis:

1. The observations on prices contain the effects of pure speculative decisions as well as the effects of the storage decisions. Speculative activity affects prices which, in turn, affect the amount of stocks carried over. Therefore, speculative activity is affecting the estimated relationship between inventory positions and the price spread.
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2. The price spread can only exist between a known future price and the cash price. The known future price only exists for those dates for which a futures contract is traded. A price spread constructed between all future prices and the cash price would be preferable, but the market does not provide this information.

3. The Chicago Mercantile Exchange delivery specifications state deliverable stocks have to be stored after December 1, in order to be deliverable at par during the next year. The inventory figures do not specify the quantity of new bellies and old bellies. Thus, the inventory figure may not be representative of the amount deliverable at par. Certainly, the old bellies are not held for delivery against the futures contract at the cessation of trading of the August contract, yet the quantity of inventory is specified as being related to the price spread which contains the future price observation. Ideally, researchers prefer to relate the quantities and price spreads of equivalent variables.

4. The measured inventory can be delivered against any contract during the year. These data do not allow identification of the amount of inventory being held to a certain future date in response to a certain price spread that exists between the two dates. Ideally, the number of pounds of inventory being transferred to a certain date in the future, and the price spread which exists between the two dates would be preferred.

These deficiencies in the data are referred to as measurement errors. The price-spread model requires certain narrowly defined data but the data which must be collected contain information in excess of what is required. As a consequence, the price-spread model is being used to test hypotheses with data that contains the effects of speculation and does not relate quantities and price spreads with a fixed time dimension.

The price-spread model hypothesizes that the amount of inventory transferred from the current time period to a future time period is a function of the price spread between the two time periods and the cost of storing the commodity between the two time periods. The model was expanded to include lagged-price spreads and lagged-quantities of inventory as independent variables. The ordinary least-squares technique was employed to estimate a linear function between inventory and the independent variables for each of the 27 subperiods that existed during the time period for which data were collected.

The statistical tests undertaken on the 27 estimated equations reject 6 as being the correct functional form to explain the variation in the dependent variable. The rejected subperiods are March, July, and August, 1965, August, 1968, and February and March, 1969. These rejections imply that the price-spread model is not an adequate theoretical model.
for explaining the variation in stored inventory. Other functional forms were estimated in an attempt to reformulate the relation, but in all cases the F-value indicated the function was to be rejected. Thus, the price-spread model does not provide the theoretical relation between stored inventory and the independent variables for these six subperiods.

The supply-of-storage theory hypothesizes a positive relationship between changes in price spreads and changes in stored inventory. Nine of the 27 subperiod equations produce a negative total change in stored inventory resulting from a unit change in the price spread. Five of the nine subperiods are the same subperiods whose estimated equations have been rejected earlier on grounds of an insignificant F-value. There remain four subperiods whose estimated supply-of-storage function is perverse.

Perverse supply curves can exist in economic theory. The theory of the supply of storage does not hypothesize a negative relation between stored inventory and the independent variables. There are only two justifications for the negative relationship. First, speculative activity dominates the formation of the futures price which affects the measured price spread. Second, the errors in measurement include the effect of speculative activity. These two reasons combined tell us the price-spread model is incomplete. A function needs to be designed to account for the effect of speculation on the futures prices, allowing the statistical procedures to separate the speculative effects on futures prices from the effects of price spread on the quantity of stocks carried over. In the hypothesized model, both effects are comingled and separation is impossible, so the equations should be rejected as representing the appropriate supply-of-storage functions.

Twenty-seven equations were originally estimated and 10 have been rejected. The remaining question is: What information is provided about the supply of storage for frozen pork bellies from the remaining 17 equations? The supply of storage is generated by decision makers observing a set of current and past price spreads and the inventory positions lagged one period. The decision process engaged by individuals who supply storage is complex, as observed by the absence of any perceivable pattern in which the lagged-price spreads enter into each of the estimated equations. If some order of entry could be detected, one could hypothesize that the decision makers first examine this price spread, second, another price spread, and so on. Since there is no apparent order, the conclusion can be drawn that while price spreads do explain the amount stored as the theory states, the process determining which price spreads influence the storage decision is unknown.

In addition, the absence of any pattern in which prices enter the decision process is confirmed by the lack of any pattern for the size of the
regression coefficients as the supply functions are estimated sequentially. This suggests two conclusions:

1. The market process is dynamic and stable. Being dynamic means that the equilibrium price spread and inventory position is constantly changing as the supply-and-demand curves for each market time period shift. Stability implies that once the market is in a disequilibrium position the market moves back towards a new equilibrium position. The perverse shape of some of the estimated supply functions indicates a temporarily unstable market, but this never exists longer than two consecutive subperiods.

2. If during any subperiod of any one futures contract a perceivable pattern is observed, the market participants will react to the pattern, and thus eliminate it. All patterns will be eliminated by the actions of the market participants with the consequence of the preceding observation.

Examination of the coefficient of determination indicates that approximately one-half of the total variation in the amount held in storage can be explained by the price spreads and lagged-inventory positions. This is an acceptable result considering the effect of speculative activity on price spreads which cannot be isolated on this model, and the deficiencies in the data. The results also imply that Samuelson's theoretical model can be empirically verified with these data.

An additional insight into the effects of speculative activity can be gained by observing the $1 - \lambda$ values and the coefficient of determination for those contract maturity months which traditionally attract speculation, and those maturity months which do not typically receive speculative interest. Contracts maturing in May receive the least speculative activity. The May equations first estimated produce an extremely high $R^2$ value implying that the price-spread model is almost a perfect theoretical base for the supply-of-storage function. In the remaining contract maturity months which attract speculative activity the $R^2$ values decline as expected since the price-spread model does not attempt to consider the effects of speculation on the amount of inventory transferred to a future time period.

The $1 - \lambda$ values can be interpreted as an indicator of the speed at which the system returns to equilibrium after a one-unit change in the price spread. This coefficient for the May equations indicates that the time required to restore equilibrium is longer than for the other contract maturities. The absence of speculative activity in this month indicates that the market's accumulation and disposal of bellies requires a considerable number of weeks before a complete inventory response can be observed, raising the perplexing and yet unresolved question: Why does the presence of speculative activity, as observed by low $R^2$ values and an a priori knowl-
edge of the months which attract speculation, produce a short time period in which inventory positions respond to changes in price spreads, while the absence of speculation produces a longer time period required for inventory positions to respond to changes in price spreads?

The estimated supply functions show that the process generating the amount of stocks carried over from one time period to the next is quite complex. The independent variables enter the sequence of equations in some indeterminate order, and the regression coefficients assume values of an unpredictable magnitude. These two observations suggest that there is not one supply function which is representative of the supply of storage for frozen pork bellies. Each of the estimated functions does indicate that some set of past price spreads is the appropriate group of independent variables.

The influence of the lagged-dependent variable is of particular importance. The rate at which equilibrium is restored depends upon the month in which the futures contract matures. The May supply function requires a longer period than the other four months and this has also been observed for the six May subperiods used in the analysis. The process generating the supply functions differs between May and the other subperiods, but this process generating the May equations has been more uniform throughout the data period than any of the other subperiods.

REFERENCES


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