MULTIPLE PRODUCT FORMS
AND SPATIALLY SEPARATED MARKETS

It became apparent in Chapter 9 that under competitive conditions the prices of all finished products derived from a common raw material must come into a particular equilibrium reflecting product yields and processing costs. If the price of one product form (or a set of products and by-products) falls below this equilibrium level, it will be unprofitable for firms to sell the raw material in that form and, hence, the product will not be produced. On the other hand, if a product price is above the equilibrium level, the raw material will be diverted from all other uses into this use. Such diversions of a raw product continue until prices are forced into perfect alignment through processing costs and equal net values for the raw material. Just as with spatially separated markets, the equilibrium mechanism is arbitrage; but here the arbitrage takes place in relation to alternative product forms.

It also became clear from our dairy products example that a given commodity will often yield products that differ widely in concentration, bulkiness, and value per pound. Thus, alternative uses of a hundred-weight of raw milk included 5 pounds of butter worth 68 cents per pound, 10 pounds of cheese worth 40 cents per pound, 20 pounds of cream worth 17 cents per pound, or 47 pounds of evaporated milk with a market price of less than 11 cents per pound. It follows that transportation costs
have a differing impact on each of the alternatives and that the simultaneous consideration of alternative product forms and the space dimension of price will establish an optimum pattern of production organization around a given market.

10.1 NET VALUES OF THE RAW PRODUCT

We begin our discussion of a market in form and space by first considering a very simple model where we neglect processing costs and by-product values. Suppose that we consider whole milk, light cream (20 percent butterfat), and butter as the only alternative uses. The first step is to find a meaningful way to compare the different market prices. For this purpose, we calculate an equivalent raw product price \( R_t \) for each by multiplying the market price \( P_t \) times the yield of the \( i \)th product per unit of raw product (in this case one hundredweight) \( k_i \). In our example these equivalent raw product prices are \( 100P_m \), \( 20P_c \), and \( 5P_b \) for milk, cream, and butter, respectively.

Now we can compare the profitability of these alternatives at any location by expressing each in terms of the net value per unit of the raw product. Under our simplifying assumptions, this involves only the market price of each form, the product yields, and the transfer-cost function.

The net value of the raw product in each form can be written as follows:

\[
N_m = 100P_m - 100(tD) \\
N_c = 20P_c - 20(tD) \\
N_b = 5P_b - 5(tD)
\]  

(10.1)

where \( m, c \) and \( b \) = the milk, cream, and butter alternatives.

\( N \) = the net value per hundredweight of raw product at a given location

\( P \) = the product price per pound at the market

\( D \) = the distance from the given location to market

\( t \) = the transport cost per pound-mile of raw product.

The constants 100, 20, and 5 represent the several product yields in pounds per hundredweight of raw milk. We assume that transfer cost increases with distance but is identical per pound-mile for all products. Here we use a linear relationship with distance \( tD \), but it is understood that transfer cost will normally increase with distance at a decreasing rate and that the transfer functions may differ for the several products.
In more general terms, we might write these net values as

\[ N_i = k_i P_t - k_i t_i D \]  

(10.2)

where \( k_i \) = yield of the \( i \)th product per unit of raw product

\( P_t \) = the market price

\( t_i \) = transport cost per pound-mile for the \( i \)th product.

The reader will recognize these functions as similar to the single-product examples developed in Chapter 7. In each case, we visualize a geographic structure of net values or "at-farm" prices for the raw product, centered on the market and declining in all directions by an amount equal to transfer costs. Moreover, this geographic structure of net values will fall off rapidly in the case of a bulky product because of the relatively large quantities to be transported but very slowly for concentrated products where the total weight to be transported per unit of raw product is small.

These net value or equivalent price cones are illustrated by the cross-section diagram at the top of Figure 10.1. Here we show the fluid milk

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**FIGURE 10.1** Efficient milk, cream, and butter zones around an isolated market.
price as high at the market center but declining rapidly with distance. Cream prices per hundredweight of raw product equivalent are lower but decline with distance at a more gradual rate. The market price for butter (again in terms of the equivalent raw product) is lowest of all, and for this concentrated product the geographic price structure is nearly flat.

10.2 IDENTIFICATION OF PRODUCT BOUNDARIES

Given market prices and transport costs, it is apparent that shippers near the market will choose to ship whole milk since the net value obtained from this use is higher than other alternatives. Farther from market, the cream alternative will be most profitable, although butter will be the optimum utilization at distant points. In this way, competition will bring about a system of product zones around the market as shown at the bottom of Figure 10.1. The boundary between any two products, such as the set of points $a$ or $b$, is found where the net values of the two alternatives are equal. The boundary between the milk and the cream zones, for example, will be represented by the set of points $a$ where the following equality holds true:

$$N_m = N_c.$$  \hspace{1cm} (10.3)

By using the data from our earlier example, Equation 10.1, this can be written as

$$100P_m - 100(tD) = 20P_c - 20(tD)$$ \hspace{1cm} (10.4)

or as

$$100P_m - 20P_c = 100(tD) - 20(tD).$$ \hspace{1cm} (10.4')

In short, the boundary is found at that distance where the difference in the market values (per hundredweight of raw product equivalent) is exactly offset by the difference in transfer costs. This principle will hold true for the boundary between every pair of products.

By solving Equation 10.4 for $D$, we find that the milk-cream boundary, in terms of miles from market, can be expressed as a ratio:

$$D_{m-c} = \frac{100P_m - 20P_c}{100t - 20t}$$ \hspace{1cm} (10.5)

or, more generally, as

$$D_{i-j} = \frac{k_iP_i - k_jP_j}{k_it - k_jt}. \hspace{1cm} (10.6)$$
It is now a simple matter to allow for differences in processing costs in this formulation since we merely subtract the appropriate costs from the net value equations. If processing costs are expressed in terms of raw product equivalents, Equation 10.5 becomes

$$D_{m-c} = \frac{(100P_m - C_m) - (20P_c - C_c)}{100t - 20t}.$$ (10.7)

However, frequently it is more convenient to express processing costs in terms of cost per unit of final product, as was done earlier in Table 9.1. In this case, the milk-cream boundary would be written

$$D_{mc} = \frac{100(P_m - c_m) - 20(P_c - c_c)}{100t - 20t}$$ (10.8)

or, more generally,

$$D_{i-j} = \frac{k_i(P_i - c_i) - k_j(P_j - c_j)}{k_{it} - k_{jt}}.$$ (10.9)

A summary of the definitions used is provided in Table 10.1 to assist the reader in shifting between final product units and equivalent raw product units.

We observe here a general tendency of markets in form and space: bulky products will have relatively high prices at the market (in terms of equivalent raw product) and will be produced adjacent to the market, but concentrated products will have relatively low at-market raw product values and will be produced at a greater distance from the market. If this were not true, then there would be no location within the producing area where it would be profitable to produce the bulky product; the milk price cone in Figure 10.1 would always be below the cream price cone, and therefore the market would be unable to obtain a supply of this bulky product.

### 10.3 Role of Market Demands and Supplies

We have assumed that prices at the market were fixed and given and have proceeded to determine the optimum allocation of the producing area among the alternative products consistent with these given prices. It should be clear, however, that market prices and the allocation of product zones are, themselves, part of a multiple-price, single-market equilibrium. There exist various demands for products at the market center, each with its characteristic elasticity. The geographic production pattern for the basic raw material also will change with changes in effective at-farm
<table>
<thead>
<tr>
<th>Product Form</th>
<th>Market Price (in Final Product Units)</th>
<th>Final Product Yield per Unit of Raw Product</th>
<th>Equivalent Raw Product Price ((R_i))</th>
<th>Volume of Output of Product (i)</th>
<th>Total Variable Cost for Product (i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milk</td>
<td>(P_m)</td>
<td>(k_m)</td>
<td>(k_mP_m)</td>
<td>(V_m)</td>
<td>(C_mV_m) (c_mk_mV_m)</td>
</tr>
<tr>
<td>Cream</td>
<td>(P_c)</td>
<td>(k_c)</td>
<td>(k_cP_c)</td>
<td>(V_c)</td>
<td>(C_cV_c) (c_ck_cV_c)</td>
</tr>
<tr>
<td>Butter</td>
<td>(P_b)</td>
<td>(k_b)</td>
<td>(k_bP_b)</td>
<td>(V_b)</td>
<td>(C_bV_b) (c_bk_bV_b)</td>
</tr>
<tr>
<td>Generalized</td>
<td>(P_i)</td>
<td>(k_i)</td>
<td>(k_iP_i)</td>
<td>(V_i)</td>
<td>(C_iV_i) (c_iV_i)</td>
</tr>
</tbody>
</table>

Equalities:
- Plant volume, \(V = \Sigma V_i\).
- Output of final product \(i\), \(Q_i = k_iV_i\).
- Variable cost per unit equivalent raw product, \(C_i = c_i k_i\).
- Total variable cost for product \(i\), \(C_iV_i = c_i k_iV_i = c_i Q_i\).
- Equivalent raw product price, \(R_i = k_i P_i\).
- Contribution of \(i\)th product to net income:
  - In raw product units: \(NR_i = R_iV_i - C_iV_i = (R_i - c_i) V_i\)
  - In final product units: \(NR_i = k_i P_i V_i - k_cV_i = (P - c_i) k_i V_i = P_i Q_i - c_i Q_i = (P_i - c_i) Q_i\).
prices. Market equilibrium will then balance off supplies and demands for each product, and the supply functions (and perhaps the demand functions) will be interrelated.

In final equilibrium, the supply function for each product will involve not only the prices and transfer costs for that product but the prices, transfer costs, and processing costs for all products. The quantity of cream delivered to market, for example, will increase if we raise the cream price relative to other prices. On the other hand, it will decrease if we raise either the milk or butter price. One can visualize the effects of such price changes as raising or lowering a price cone and observe the effects on the boundaries with other price cones in the system. Similarly, changes in transfer costs or processing costs will influence not only the product directly involved but also the supplies of all products. In this important way, then, the whole structure of product prices and zones is interrelated and interdependent.

It is easy to demonstrate that in a perfect market these equilibrium product zones will result in the lowest aggregate transfer cost for all products consistent with meeting market requirements of the several products. Suppose that we consider shifting one unit of raw product located at some point $X$ in the milk zone from milk to cream use and that we compensate for this by shifting a unit of raw product from cream to milk use at any point $Y$ in the cream zone. Notice that milk is the bulky product and subject to higher transfer cost per unit of distance in terms of raw product equivalent—it costs more to ship 100 pounds of milk than 20 pounds of cream. Point $Y$, therefore, must be farther from market than point $X$. Now, the indicated shifts represent a net increase in the distance that the unit of milk is shipped and an exactly equal decrease in the distance that the unit of cream is shipped. But, since it costs more to ship the raw product equivalent as milk than as cream, the net effect must be an increase in transfer cost. This will be true for any pair of points that we choose and for any pair of products and, hence, the competitive product zones must minimize transfer costs for the aggregate market.

Not only do these product boundaries define the most efficient allocation from the standpoint of transfer costs but they also represent the maximum aggregate returns to producers consistent with perfect competition. In our example above, point $X$ is in the milk zone and this is closer to market than point $Y$ in the cream zone. We know that the net value is higher for milk than for cream at point $X$, although the reverse is true at point $Y$. Shifting point $X$ to cream would thus reduce net value, while shifting point $Y$ to milk would also reduce net value. On both scores, then, such shifts would reduce the net value of the raw product. Since net values represent producer payments (at-the-plant), clearly, the
competitive or free-choice zones permit consumers at the market center to obtain the demanded quantities of the several products at the lowest aggregate expenditure.

10.4 POINT TRADING, SPACE-FORM MODELS

To introduce the idea of two or more competing markets with several alternative product forms required in each, we use a modification of the transportation model that was discussed earlier (Chapter 5). As described there, the transportation model was a single-dimension model because shipping and receiving points were separated from each other in the single dimension of space. Now we make use of a multiple-dimension model in which shipping and receiving points are separated not only in space but also with respect to the form in which the product is delivered.

Transfer costs in this multiple-dimension model contain other charges in addition to transportation cost. The cost matrix now includes not only the cost of transfer between shipping point and destination but also the cost of transformation or processing. In this way we can simultaneously determine optimum locations for processing and interregional commodity movements.

To illustrate a two-dimension transportation model, a fluid milk example that uses six regions and two product forms is presented in Table 10.2. Surplus quantities may be moved to market from exporting regions in either conventional or concentrated form. Producers are indifferent as to the form in which the milk is sold, since the competitive market equalizes net returns from each after deduction of processing costs. Consumers, on the other hand, regard the two forms as different products. Requirements for each of the two forms are known for each of the importing regions. It is possible to allow for differences among surplus regions in the costs of converting milk into the concentrated form as well as in the proportion by which costs of transfer are reduced as a result of such processing. Transfer costs, then, for the concentrated form consist of the sum of the cost of processing and the cost of transportation, although the transfer costs for the conventional product include only the usual transportation costs.

An examination of the flow matrix in section 2a shows that the equilibrium solution to this problem contains a new item of information. By summing the quantities in the columns labeled "concentrated," we obtain an estimate of the processing capacity needed in each surplus region with the market in equilibrium. In section 2b the \( u_i \) values again indicate relative prices in the three exporting regions (before sign change), and \( v_j \)
### TABLE 10.2  Six-Region, Two-Dimension Model (Space-Form)

<table>
<thead>
<tr>
<th>Exporting Region</th>
<th>Importing Region and Form of Commodity</th>
<th>New England</th>
<th>South Atlantic</th>
<th>California</th>
<th>Surplus ($S_i$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conventional</td>
<td>Concentrated</td>
<td>Concentrated</td>
<td>Concentrated</td>
<td></td>
</tr>
<tr>
<td>Minnesota</td>
<td></td>
<td>2.70</td>
<td>3.20</td>
<td>4.00</td>
<td>0.60</td>
</tr>
<tr>
<td>Wisconsin</td>
<td></td>
<td>2.25</td>
<td>2.70</td>
<td>4.50</td>
<td>0.63</td>
</tr>
<tr>
<td>New York</td>
<td></td>
<td>0.55</td>
<td>2.60</td>
<td>8.00</td>
<td>0.75</td>
</tr>
</tbody>
</table>

#### Dollars per Hundredweight

| Deficit, $D_j$ (physical units) | 2.5 | 2.5 | 4.0 | 4.0 | 7.5 | 7.5 | 28.0 |

#### Physical Units

|---|---|---|---|---|---|---|---|---|---|

*Numbers in parentheses are optimum supply allocations in physical units.*

*bProducer price differentials before sign change.*

*cConsumer price differentials.*
entries in the same section show price differentials between each market and the base region, not only for the conventional form but also for the concentrated form. Notice that Minnesota has been selected as the base region in this example. As in the earlier example, section 2c shows the cost of using nonoptimum routes which, in this case, provide information on the added cost of locating processing facilities in certain areas as well as the cost of selecting routes that do not enter the minimum transportation cost solution.

This space-form version of the transportation model has been used in a variety of empirical studies. For example, Snodgrass and French used this model to determine optimum locations of processing facilities in a study of the national dairy products market. The trade patterns shown in Figure 10.2 are not actual trade patterns for milk in fluid form or as evaporated milk, cheese, or butter. Instead, the calculated movements among 24 regions in the United States are the ones that would minimize processing and transportation costs. Movements within regions are not included. Notice that this map clearly indicates the dominant position of

![FIGURE 10.2 The optimum interregional movements of dairy products to minimize processing and transportation costs, 1953. M, fluid milk; E, evaporated milk; C, cheese; and B, butter. All movements in terms of milk equivalent. [Source. Milton M. Snodgrass, "Linear Programming Approach to Optimum Resource Use in Dairying" (unpublished Ph.D. dissertation, Department of Agricultural Economics, Purdue University, 1956), Table 23, p. 99.]](image_url)
Iowa, Wisconsin, and Minnesota in the market for dairy products with product flows radiating out from this focal center. Butter is the predominant product in Minnesota and Iowa with cheese shipments intermingling with butter at the periphery of the central core. Virtually all regions other than the ones in the North Central States are deficit with respect to dairy products. Massive shipments are made to the population centers along the Atlantic seaboard, and smaller but important movements fan out to the southern and western states. Evaporated milk shipments are minor relative to butter and cheese and, for the most part, originate from the zones beyond the major butter-cheese producing areas. Interregional shipments of whole milk for fluid purposes would not be large as a consequence of bulkiness and high transfer costs, but some shipments are indicated to meet needs in New England, Florida, Texas, and the Southwest. Although the complete details of actual movements are not available to compare with these calculated and efficient patterns, the general organization of the industry is remarkably consistent with the model results.

10.5 COMPETING MARKETS WITH PRODUCTION DISPERSED

The full complexity of interproduct boundaries discussed in section 10.2 and intermarket boundaries considered in Chapter 9 is difficult to visualize. Therefore, to provide a more complete understanding of the price structure of multiple-form markets, we illustrate in Figure 10.3 a case with four competing markets or "dealers" identified as A, B, C, and D. Each of these markets receives milk and cream from the surrounding production area, with the most distant area serving as the supply area for manufactured dairy products. A variety of transportation methods are used, including direct haul to the plant, the shipment of milk by tank truck and cream by rail. In Figure 10.3, we find a reproduction of the type of price cones illustrated in Figure 10.1 except that now we have multiple cones surrounding each of the four markets.

The upper section of Figure 10.3 represents the net farm price surface for alternative products. Net farm prices are shown as cones centered on each market. The lower section of the diagram delineates the product supply areas for each of these markets. Sufficient milk is provided to each market to meet their fluid milk and cream requirements, with any remaining supplies sold in manufactured form. It is assumed that this is a deficit area with regard to manufactured dairy products so that it is not necessary to consider equilibrium prices for milk other than that needed for milk and cream purposes. Plants that ship fluid milk or cream
are located nearer to the markets of this region and pay a higher price to induce farmers to shift from selling milk to local manufacturing plants.

Markets B and D are the major markets in the region. Both compete for the same milk supply areas. Since the demand for milk in market D is large and D is the most distant from the surplus-milk producing area (being located on the edge of the region), prices in this market are higher than in market B. The farm price surface is higher around D than around B to permit both the milk and cream supply areas of D to encroach on the supply area of B. In this example, the milk supply area for market D extends approximately 330 miles with the line UV forming the market boundary with the milk supply area of market B. The cream supply area of market D is in contact with both the milk and cream supply areas of market B along the boundary VQW. The cream supply area of D extends 550 miles, whereas that of B extends 350 miles.

The milk supply area of market C, a small secondary market, is completely surrounded by the milk supply area for D. The price of milk in C is lower than but is directly related to the milk price in D. On the intermarket boundary, the milk producer chooses between shipping direct to the city plant at C or selling to a country receiving plant which in
turn ships milk to D. The cream requirements of market C are obtained from the same general area as the cream supply area of D.

These intermarket boundaries could now be defined in algebraic form as was done earlier for product boundaries. Clearly, it would be necessary to add a subscript referring to market as well as a subscript identifying the product form. This, however, will be left to the interested reader to do for himself. The theory is clear—so long as the net price for a particular product form in one market exceeds that of all other forms in that market as well as that for every form in all other markets, producers will choose the first alternative. Whenever it becomes profitable to change form or to shift to another market, the producer will do so under competitive conditions. The set of points at which net prices of products shipped to any two markets are equal will form the intermarket boundaries shown in Figure 10.3. In some cases, for instance, in the case of market boundary UV, the market boundary will divide a given product zone, such as the area engaged in the shipment of fluid milk by tank truck. In other cases, however, such as the eastern edge of the milk market around A, the boundary will separate fluid milk shipped to one market, A, from cream shipped to another market, B.

An empirical example of the usefulness of this model is provided in Figure 10.4. Because of substantial differences in milk supplies in fall and
spring months, it was necessary to calculate the shape of the several milksheds for these two periods separately. At the time that the study was made, very nearly the entire supply of milk produced during the fall months was shipped as fluid milk. However, with heavier production in the spring, some areas shifted to cream and, in a few cases, to the production of manufactured milk products. Associated with these product zones are a set of imputed intermarket price differences that reflect demands in each market and the availability of supplies to meet these demands.

We now repeat certain generalizations concerning interrelated market prices. A change in the price of any product form in any market will influence not only the supply areas for other forms of that product in the given market but will also have effects on the boundaries between that market and every other market in the interconnected system.

10.6 SEASONAL VARIATION IN PRODUCT BOUNDARIES

Recognizing that demands and supplies vary seasonally for many products, we now inquire in more detail into the effects of such changes on plant operations. As suggested in Figure 10.4, seasonal supply and demand changes give rise to seasonal changes in product prices; and they, in turn, affect the boundaries between product zones. With a boundary between two product zones that varies from month to month, the result must be one zone that is specialized throughout the year in the shipment of a product such as milk, a more distant specialized cream zone, and a third intermediate, diversified zone that sometimes ships milk and sometimes cream. The general outlines of these zones are suggested in Figure 10.5. Our earlier illustration drawn from a study of northeastern milk markets shows that these diversified zones are not simply a theoretical possibility. Figure 10.6 identifies the geographic areas where the most efficient use of raw milk changes from season to season.

We now investigate the particular problems posed for plants located in these diversified regions. Let us consider the situation confronting the manager of a plant located in the milk-cream diversified zone, noticing that the general findings for this location are appropriate for other two-product diversified zones. We assume that the market is characterized by perfect competition. We also assume that managers act intelligently in their own self-interest and are not misled by some common accounting folklore with respect to fixed costs (although this is more a warning to our readers than a separate assumption, since it is implicit in the assumption of a perfect market).
FIGURE 10.5  The specialized and diversified product zones resulting from seasonal supply and demand fluctuations.

FIGURE 10.6  The efficient seasonal changes in milk, cream, and manufacturing zones for major northeastern milk markets, 1947 to 1948. [Source. William Bredo and Anthony S. Rojko, Prices and Milksheds of Northeastern Markets].
We assume that the plant in question serves a given number of producers located near the plant and that this number is constant throughout the year. Production per farm varies seasonally, however, so that even under ideal conditions the plant will handle volumes less than capacity during the fall and winter months. We assume that the plant is equipped with appropriate facilities so that it can divert its entire volume to milk or to cream shipment. Market prices for milk and cream vary seasonally, and to meet demands in the low-production period, milk prices change more than cream prices. With given plant location and transportation costs to market, this means that the manager is faced with seasonally changing milk and cream prices f.o.b. his plant. Our problem is to indicate the effects of these changes on plant output.

10.7 SELECTION OF PRODUCT FORM

Consider first the cost function for a plant. We assume that variable costs are constant per unit of output and that output of either product can be expanded at the specified variable cost per unit up to the limits imposed by plant equipment (capacity) and available raw product. This constant variable cost assumption simplifies the presentation and is, in fact, a good approximation to the situation in many plants.

In addition to variable costs, the plant operation involves certain fixed or overhead costs. They are independent of the volume of the several products but reflect the particular pattern of plant facilities and investments. As far as these fixed costs are concerned, the several outputs must be recognized as joint products. Although there are a number of ways in which these fixed costs might be allocated among the joint products, all are arbitrary.

Fortunately, such allocations of fixed costs are not necessary to the determination of firm policy concerning the selection of optimum production patterns. In fact, fixed-cost allocations would only serve to confuse the issue. We take fixed costs as given in total for the year, although even this is arbitrary, for the outputs of any two years are also joint products. The pertinent idea is that the firm should recover its investment over appropriate life periods. If it does not, it will not continue to operate over the long run; if it more than recovers investments plus associated costs, then the abnormal level of profits will induce the entry of new firms and return profits to the normal level. Many of the usual fixed costs are institutionally connected with the fiscal year, however, and for this reason the treatment of total fixed costs on a yearly basis seems appropriate.
DIMENSIONS OF MARKET PRICE

Examples include annual interest charges, annual taxes, and annual salaries for management and key personnel.

In terms of total costs per year, we visualize a surface corresponding to a function of the type

\[ TC = A + C_1 V_1 + C_2 V_2 \]  \hspace{1cm} (10.9)

where

- \( A = \) annual fixed costs
- \( V_1 \) and \( V_2 = \) annual output of each product
- \( C_1 = \) variable cost per unit for product 1
- \( C_2 = \) variable cost per unit for product 2.

all expressed in terms of the raw product equivalent. If we wish to express total costs in terms of the final products, we would write Equation 10.9 as

\[ TC = A + c_1 k_1 V_1 + c_2 k_2 V_2 \] \hspace{1cm} (10.9')

where \( c_1 \) and \( c_2 \) are variable costs per unit of final product, and \( k_1 \) and \( k_2 \) are final product yields per unit of raw product.

Total revenue to the plant is represented by product outputs multiplied by f.o.b. plant prices, or

\[ TR = k_1 (P_1 - t_1 D)V_1 + k_2 (P_2 - t_2 D)V_2 \] \hspace{1cm} (10.10)

where \( P \) and \( t \) represent market prices and unit transport costs—again multiplied by \( k_1 \) and \( k_2 \) to convert to raw product equivalent terms. Net returns, or the net value of the raw product, is represented by total revenue minus total costs. or

\[ NR = TR - TC \] \hspace{1cm} (10.11)

where

\[ NR = k_1 (P_1 - t_1 D)V_1 + k_2 (P_2 - t_2 D)V_2 - A - C_1 V_1 - C_2 V_2. \]

If the manager wishes to maximize his net return (and under perfect competition he has no alternative if he is to remain in business), then he can calculate his optimum adjustments by using familiar methods. To determine the net-revenue maximizing adjustments, we take partial derivatives of the net-revenue function. Equation 10.11, with respect to each product, set each equal to zero, and solve:

\[ \frac{\partial NR}{\partial V_1} = k_1 (P_1 - t_1 D) - C_1 = 0 \] \hspace{1cm} (10.12)

\[ \frac{\partial NR}{\partial V_2} = k_2 (P_2 - t_2 D) - C_2 = 0. \]
We may substitute for $C_1$ and $C_2$ in Equation 10.12 the values $c_1 k_1$ and $c_2 k_2$. We also substitute the f.o.b. plant prices $P'_1$ and $P'_2$ for the corresponding market prices less transfer cost. The partial derivatives may now be written

$$\frac{\partial NR}{\partial V_1} = k_1 P'_1 - k_1 c_1 = 0$$

$$\frac{\partial NR}{\partial V_2} = k_2 P'_2 - k_2 c_2 = 0.$$  \hspace{1cm} (10.13)

This merely confirms what we already know—that to maximize net returns, the manager should expand each line of production so long as f.o.b. prices exceed marginal processing costs. Notice that this optimum decision is in no way dependent on fixed costs or on any arbitrary allocation of these fixed costs.

But, under the assumed conditions, marginal costs are constant; and, at any time, f.o.b. plant prices are also constant. Apparently, the above equations do not define equilibrium adjustments involving the combined output of the two products. Instead, they tell the manager not to produce a product if price is less than marginal cost and to produce to the limit of plant capacity and available raw product if price exceeds marginal cost. For any set of prices, one or the other product will be more profitable, and the entire output of the plant will be in the more profitable alternative. In algebraic terms, we state the following rules for the manager.

If $(P'_1 - c_1)k_1 > (P'_2 - c_2)k_2$, ship only product 1. 
If $(P'_1 - c_1)k_1 < (P'_2 - c_2)k_2$, ship only product 2. 
If $(P'_1 - c_1)k_1 = (P'_2 - c_2)k_2$, ship either 1 or 2.  \hspace{1cm} (10.14)

Since we have assumed that this plant is located in the milk-cream diversified zone, it follows that prices will make milk shipment the more attractive alternative during part of the year, and cream will be more profitable in other seasons.

We have assumed that product prices will vary seasonally and have observed that milk prices will vary more than cream prices. We have also assumed that the marginal or variable unit costs $c_1$ and $c_2$ are given and constant. We may restate the above profit-maximizing conditions as rules for the manager.\(^1\)

\(^1\)This is done by equating the partial derivatives in Equation 10.13, dividing through by $k_2 P'_2$, substituting to eliminate the two terms on the right-hand side, and finally multiplying the remaining terms by $k_2/k_1$. 

When \( \frac{P_1'}{P_2'} > \frac{c_1}{c_2} \), ship product 1.

When \( \frac{P_1'}{P_2'} < \frac{c_1}{c_2} \), ship product 2. \hspace{1cm} (10.15)

When \( \frac{P_1'}{P_2'} = \frac{c_1}{c_2} \), ship either product.

The manager will watch the ratio of f.o.b. plant prices and compare them with his marginal cost ratio. If the price ratio always exceeds the cost ratio, the plant should always ship whole milk and, thus, be in the specialized milk zone. If the price ratio always is lower than the marginal cost ratio, optimum operation calls for specialized cream shipments at all times. But if this plant is, in fact, located in the diversified milk-cream zone, then during some of the fall and winter months the price ratio will exceed the marginal cost ratio and the plant will ship milk. But during the spring and summer months, milk prices will decline relative to cream prices, the price ratio will fall below the marginal cost ratio, and cream shipment will be more profitable. Day by day and week by week the manager will make these decisions, and the result will be a particular seasonal pattern of milk and cream shipments. If the plant is located near the inner boundary of the milk-cream diversified zone, then it will ship milk during most of the year and cream for only a few days or weeks during the peak production period. Conversely, a location near the outer boundary of this zone will dictate cream shipments during most of the year with milk the more profitable alternative for a short period when production for the market is very low.

10.8 LONG-RUN ADJUSTMENT TO SEASONAL VARIATIONS

It may be protested that the foregoing analysis is incorrect because a plant that uses its separating equipment for only a few days must have very high cream costs. This is a common misunderstanding; it arises from the practice of allocating fixed costs to particular products. Nevertheless, a grain of truth is involved, and it can be correctly interpreted by considering the alternatives of specialized milk plant or milk-cream diversification near the milk and milk-cream boundary.

We have learned that the net value of raw product for the diversified plant can be represented by

\[
NR_d = k_1 P_1' V_1 + k_2 P_2' V_2 - A_d - C_1 V_1 - C_2 V_2. \hspace{1cm} (10.16)
\]
In a similar way, we represent net values for the specialized milk plant as

\[ NR_s = k_1 P'_1 V_1 - A_s - C_1 V_1 \]  

(10.17)
in which \( A_s \) represents the fixed costs for a specialized milk plant and \( C_1 \) the variable costs. We assume that the variable costs of shipping milk are the same in the two types of plant, although this may not be true and is not essential to our argument.

In our equations, prices are given in terms of the milk equivalent of the whole milk or cream and are expressed at country plant location. By remembering that the at-plant price is market price less transportation cost to market and that transportation costs are functions of distance, we can use these costs to define the economic boundary between the specialized milk plant zone and the transition milk-cream zone. For simplicity, we represent the transportation costs by \( t_1 D \) and \( t_2 D \) and give the expression for the distance to the boundary of indifference below:

\[ D_{d-s} = \frac{(k_1 P_1 - C_1) - (k_2 P_2 - C_2) + (A_d - A_s)/V_2}{t_1 - t_2}. \]  

(10.18)

Notice that this boundary is long run in nature; it defines the distance within which it will not be economical to provide separating facilities but beyond which plants will be built with these facilities.² The short-run situation would be represented by the margin between specialized milk shipment and diversified milk-cream shipments where all plants are already equipped to handle both products. From the material given earlier, it is clear that the equation for the short-run boundary will be exactly the same as the long-run equation, except that the fixed costs term \( (A_d - A_s)/V_2 \) will be eliminated. From this it follows that the long-run boundary will be farther from market than the short-run boundary. If a market has reached stable equilibrium, separating facilities will not be provided until a substantial volume of milk can be separated.

The actual determination of these boundaries will depend on the specific magnitudes of the several fixed and variable cost coefficients, the patterns of seasonal production, the relative transfer costs, and the patterns of seasonal price changes. Ideally, all of them interact to give a total equilibrium for the market. We may illustrate the solution, however, by assuming values for the various parameters and seasonal patterns. This has been done, with the results shown in Figure 10.7. Here we have

²We assume that equipment will have adequate capacity to handle total plant volume. The possibility remains that a plant would provide some equipment for a particular product but less than enough to permit complete diversion. As equipment investments and operating costs normally increase less rapidly than capacity, it usually will pay to provide equipment to permit complete diversion of plant volume if it pays to diversify at all.
assumed that fluid milk prices change seasonally; the prices minus unit variable costs at country points are represented by line $ab$ for the high-price season and line $cd$ for the low-price season. We have assumed that cream prices are constant. Although this is not strictly correct, it will permit us to indicate the final solution in somewhat less complicated form than otherwise would be necessary. The geographic structure of cream prices less direct variable costs is represented by line $cb$. Apparently, the short-run boundary between the specialized milk zone and the milk-cream zone would be at distance $on$, since at point $c$ net raw product values would be equal in either alternative. Similarly, the outer short-run margin between the milk-cream zone and the specialized cream zone would be at distance $os$.

Consider the long-run situation where decisions as to plant and equipment are involved. For convenience, express all net values in terms of the averages for the entire year. The net value of raw product from specialized milk plants is represented by line $ef$. This line is a weighted average of lines such as $ab$ and $cd$, each weighted by the quantity of milk handled at that particular price. The line represents the seasonal weighted-average price minus direct variable cost and minus annual average fixed costs $A/V$ per unit of raw product. In other words, this net value line is long run in that it shows the effects of fixed costs as well as variable costs and seasonal price and production changes. Similarly, line $gh$ represents...
long-run net value of raw product in specialized cream plants differing from $cb$ by the subtraction of average fixed costs $A_d/V$. Apparently, the economic boundary between specialized milk and specialized cream plants would be at point $t$ if we prohibited diversified operations. But we know that plants equipped with separators would find it economical to diversify seasonally in zone $ns$.

The increase in net value realized by cream plants through seasonal milk shipments is represented by the curved line $jkm$ in the diagram. As we start at point $m$ on the outer boundary of the diversified zone and move to plants located closer to market, an increasing proportion of the raw product during any given year will be shipped to market as whole milk. These milk shipments occur during the low-production season, as milk prices are then at their highest levels. Observe that these plants are covering total costs, including the costs for fixed separating equipment, even though a smaller and smaller volume of milk is separated. That is, the dominant consideration in this situation is the opportunity for higher net values through milk shipments — and not higher costs based on an arbitrary allocation of certain fixed costs to a diminishing volume of cream. Notice also that, under competitive conditions, plants must make this shift to milk shipment. Otherwise, they could not compete for raw product and, hence, would be forced out of business.

Although plants equipped with separating equipment would find it economical to ship small volumes of cream in the low-price period, even from the zone $nr$, the gains would not be adequate to cover the long-run costs of supplying separating equipment. This means that specialized milk plants (without separating equipment and so with lower fixed costs) are more economical in this zone. This is indicated by the fact that line $jkm$ falls below the net value line $ef$ for specialized milk plants in the $jk$ segment. The boundary specified by our long-run equation is found at distance $or$, where net long-run values are equal to $rk$ for both specialized milk plants and for diversified plants. Plants at this boundary would find it economical to ship cream for a month or two each year if they shipped cream at all. This abrupt change from specialized milk plants to plants shipping a fairly substantial volume of cream is a reflection of the added fixed costs, and this represents the previously mentioned grain of truth in the usual statements about the high plant costs involved in shipping low volumes of cream or similar products.
SELECTED READINGS

Multiple Product Forms and Spatially Separated Markets


