The Value of Information to Hedgers in the Presence of Options

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A large industry exists to provide market outlook information to producers of agricultural commodities. Producers buy this information expecting that they will be able to sell their production at more favorable prices and receive guidance in choosing from a wide array of marketing alternatives. The availability of options on futures greatly increases the opportunities for producers to capitalize on information (Adam; Bullock and Hayes).

However, the availability of options also increases the complexity of the marketing decision since, in addition to expectations of price mean, expectations of price volatility are important. Black and Scholes showed that sizable profits could be earned from incorporating perfect forecasts of volatility into their option pricing model. In a hedging context, Hauser and Andersen also noted the importance of forecasting variance in determining appropriate options strategies, and showed that a variance expectation different than the market’s implied variance affects the relationship between a trader’s expected risk and return.

The literature contains numerous attempts to forecast the mean of prices and, more recently, their variance (e.g., Feinstein; Aradhula and Holt). Some success has been reported in generating models that forecast price mean better than do futures prices, even with relatively simple models (e.g., Garcia et al.; Leuthold et al.). Fewer successes have been reported from attempts to build variance forecasting models, as building such models is more difficult. Little effort has been devoted to measuring the relative value of mean and variance forecasts.

The value of improved forecasts can be assessed in the context of decisions made by producers. The value of information is generally calculated as the difference between the expected utility from using the information compared to the expected utility generated without the information, evaluated under the probability distribution corresponding to the better information (Antonovitz and Roe (1986); Byerlee and Anderson; Babcock). In this context, limited research exists on the value of improved forecasts of the expected mean and volatility of prices when producers have the opportunity to take positions in both futures and options markets.

The purpose of the paper is to assess the relative value of better forecasts of the mean and volatility of hog prices for making producer marketing decisions. The producer’s decisions are modeled in an expected utility framework in which marketing strategies consisting of futures contracts, and put and call options at various strike prices can be chosen based on the producer’s assessment of the distribution of prices. Futures and options prices provide a market forecast of the mean and variance of hog price. A producer who has

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information that the true distribution differs from this market forecast can maximize expected utility by altering the choice of marketing instruments. The change in expected utility represents the value of the information to the producer. Here, a simulation framework is used to identify the value of better forecasts to the producer under alternative price distribution scenarios. A flexible form regression is estimated to identify the relative value of the mean and volatility forecasts. The findings indicate that mean forecasts are of higher value than volatility forecasts, and that improved volatility forecasts provide greatest additional value when combined with improved mean forecasts.

The Theoretical Model


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Producer Model

A two-period model is used to simulate a hog producer's choice of pricing strategies. In period 1, given a quantity of the cash commodity which in period 2 will equal the size of a futures contract, the producer formulates an assessment of the bivariate distribution of cash and futures prices. Expected utility is maximized by buying or selling puts, calls, and futures contracts. These contracts are offset at the time the cash commodity is sold.

Income (R) is represented as the sum of cash sales and profits made in futures and options markets. Formally, R is

\[
(1) \quad R = Qy + \sum_i [p_i^2 - rp_i^1]NP_i + \sum_i [c_i^2 - rc_i^1]NC_i + \left[ f^2 - f^1 \right]NF
- \left( t_{o_i^2} + rt_{o_i^1} \right) \text{abs}(NP_i) - \left( t_{o_i^2} + rt_{o_i^1} \right) \text{abs}(NC_i) - \left( rtf \right) \text{abs}(NF),
\]

where:

- \( R \) = income
- \( Q \) = quantity of cash commodity to be sold in period 2
- \( y \) = price per unit of cash commodity in period 2
- \( r \) = risk-free rate of return + unity (r adjusts period 1 premium and commission values to period 2 terms)
- \( p_i^t \) = price of put option at jth strike price in period t, t = 1, 2
- \( c_i^t \) = price of call option at ith strike price in period t, t = 1, 2
- \( f \) = price of futures contract in period t, t = 1, 2

NF, NP_i, and NC_i are integers representing contracts in futures, puts at the jth strike price, and calls at the ith strike price (positive values indicate long positions in period 1; negative values indicate short positions); \text{abs} indicates the absolute value of the integer contracts. \( t_{o_i^1} \) is the transaction cost for put options at the jth strike price in period t, \( t_{o_i^1} \) is the transaction cost for call options at the ith strike price in period t, and \( tf \) is the transaction cost for the futures contracts.

In this framework, the producer’s problem is:
(2) \[ \text{Max } EU(R) \]
\[ \text{NF, NP}_j, \text{ NC}_i \]
\[ \text{s.t. } \text{NF, NP}_j, \text{ and NC}_i \text{ are integers} \]
or,

(3) \[ \text{Max } \int U(R) G'(R)dR \]
\[ \text{NF, NP}_j, \text{ NC}_i \]
\[ \text{s.t. } \text{NF, NP}_j, \text{ and NC}_i \text{ are integers}, \]

where \( U(R) \) is the producer's utility function and \( G'(R) \) represents the producer's assessment of the probability density function of \( R \).

Value of Information

Futures and options prices provide a market forecast of the mean and variance (implied by the options premium) of hog price. A producer who has acquired information that the true distribution differs from this market forecast can maximize expected utility by altering the choice of hedging instruments from those chosen under market information. When the information is perfect (the producer knows the distribution that will hold for period 2 prices, but not the actual prices), the ex ante value of the forecast information \((V_1)\) is calculated as

\[ E_p U(W(X_2)) = E_p U(W(X_1) + V_1), \]

where \( W(X_i) \) is the wealth generated by strategy \( X_i \), the strategy chosen under \( E_1 \), market information, and \( W(X_2) \) is the wealth generated by strategy \( X_2 \), the optimal strategy chosen under \( E_p \), a correct assessment of the price distribution (Antonovitz and Roe (1986)). Certainty equivalents are used to express the value of information in money terms:

\[ V_1 = CE_p(X_2) - CE_p(X_1), \]

where \( CE_p \) is the certainty equivalent calculated under correct information.

When a producer acquires more accurate (less biased), but not necessarily perfect, information, the problem is more complex. The value of the forecast information \((V_2)\) is calculated as

\[ E_p U(W(X_2)) = E_p U(W(X_1) + V_2), \]

where \( E_p \) is the correct assessment of the price distribution, \( W(X_i) \) is the wealth generated by the strategy chosen under market information, and \( W(X_2) \) is the wealth generated by a less biased assessment \( E_2 \). In certainty equivalent terms,

\[ V_2 = CE_p(X_2) - CE_p(X_1), \]
where $CE_p$ is the certainty equivalent calculated under the correct assessment of the price distribution, $X_1$ is the strategy chosen under market information, and $X_2$ is the strategy chosen using a more accurate forecast of mean, volatility, or both.

The next section applies the above model to a hedging\(^1\) simulation in which a producer of a nonstorable commodity (e.g., hogs) chooses a marketing strategy in period 1 that is held until period 2 when the cash commodity is sold.

**Empirical Specification**

*Producer Model*

Following Wolf, and Hanson and Ladd, only price risk on a fixed quantity is considered. Given the confinement technology used in hog production, quantity risk is assumed to be minimal. The hog producer is assumed to farrow an amount of pigs in period 1 whose sale weight six months later (in period 2) will equal the size of a futures contract. Six months is the approximate lag between farrowing pigs and selling them for consumption. It is assumed that no trades take place between period 1 and period 2, so that the producer is required to make only one hedging decision. Options and futures contracts are offset at the time the cash commodity is sold, and no time value remains in the option premiums.

The commission costs of using futures and options contracts are considered in evaluating marketing alternatives. Here, the commission cost for futures is $80/contract per round turn, or $.27/cwt. For options, it is 5% of the premium on each purchase or sale (e.g., an option with a premium of $2.76/cwt would cost $.14/cwt if the option were allowed to expire, and $.28/cwt if it were offset with another purchase or sale in the options market). The commission costs assumed are those that are commonly charged by a full-service broker to a producer who trades only one or a few contracts at a time. Since average commission costs typically decrease as the number of contracts traded increases, these costs may be higher than many producers would be required to pay. Also, because full-service quotes were used, discounts may be available. Thus, the commission costs assumed here may influence the optimal choice slightly in the direction of a cash-only marketing strategy.

Given these assumptions, producer income, $R$, expressed in $/cwt.,$ can be rewritten as

\[
R = y + \sum_i [\text{Max}(x_p - f^2, 0) - r_p^i]NP_j + \sum_i [\text{Max}(f^2 - x_c, 0) - r_c^i]NC_i \\
+ (f^2 - f^1)NF - (t_o^2 + rt_o^1)\text{abs}(NP_j) - (t_r^2 + rt_f^1)\text{abs}(NC_i) \\
- (rtf)\text{abs}(NF),
\]

\(^1\)Hedging may be defined differently in other contexts, such as for IRS purposes; hedging here denotes only that the producer has a cash position.
where $x_{p_i} = j$th strike price for put options, $x_{c_i} = i$th strike price for call options, and $t_{02}$ and $t_{03}$ are zero if the respective option is not exercised.

With an initial cash position, $Q$, the producer generates income by simultaneously choosing positions in futures and options. To make the simulation manageable, several assumptions are made about the producer's choice set. Three strike prices for puts and three for calls are considered: one at the money, one $2$ in the money, and one $2$ out of the money. Also, the producer is permitted to buy or sell only one futures contract, as well as one put and one call at each strike price.\(^2\) The number of strategies involving integer multiples of contracts is given by $3^{i+j}$, where $3$ is the number of instruments traded (i.e., futures, put, and call options), $i$ is the number of call strikes, and $j$ is the number of put strikes, and with a futures contract adding an additional combination. This means that $2,187$ marketing strategies ($3^3$) are permitted under each assessment of the ending price distribution.

Under these assumptions, expected utility can be written

$$EU(R) = \sum \int \int U(R) L'(y, f^2) dy df^2,$$

where $L'(y, f^2)$ is the producer's assessment of the joint distribution of cash price and futures price, $LF^2$ and $UF^2$ are the lower and upper bounds of integration for the futures price, and $LY$ and $UY$ are the lower and upper bounds for the cash price.

**Structure of the Simulations**

Forty nine sets of price assessments are considered, based on a scenario in which mean and volatility reflect prices and their variation for the 1980-88 period. In this base scenario, the current (period 1) futures price for the contract expiring six months later (period 2) is $44/cwt and is used as the producer's assessment of the mean of the price distribution. Also, the producer's assessment of the annualized percentage standard deviation of log-price returns in period 2 is 23, which reflects the annualized average six-month volatility of the futures contract. In other scenarios, the producer's assessment of the mean varies in increments from $40/cwt to $48/cwt, a range of 9% in either direction from the market's forecast. Consistent with the variability of annualized volatilities found over this time period, the producer's assessment of volatility varies from 16 to 30, a range of 30% in either direction from the market's forecast.

Cash and futures prices are specified to follow a bivariate lognormal distribution. This formulation is based on previous research (Hauser, Andersen, and Offutt) and the results of statistical testing performed here which could not reject lognormality of daily price relatives. The expected mean of the period 2 cash price is assumed to equal the expected mean of the

\(^2\)Examination of situations where multiple futures contracts or multiple options contracts at the same strike price were most likely to occur (i.e., where producer assessments of price mean and volatility differ most from market forecasts) indicated that these one-contract restrictions were not binding.
period 2 futures price, with basis risk entering the model through the correlation coefficient of the bivariate distribution. The correlation coefficient between cash and futures is set at .95, which reflects the correlation of futures and cash prices in various cash markets (e.g., Omaha) on the last option trading day for each September and March futures contract over the 1980-88 period. The option premiums in period 1 are calculated from Black's model using a volatility of 23 and an underlying futures price of $44/cwt, which should provide representative premiums for the analysis considered here (Hauser and Neff).

Solution Procedures

One approach to analyze this situation is to rank marketing strategies with stochastic dominance criteria or a particular specification of an expected utility model (e.g., Schroeder and Hayenga). Most of these studies have used historical data to determine what the best strategies would have been at a particular time, given a particular production process. Often, such studies are limited by the characteristics of the time period studied.

An alternative approach examines the comparative statics of optimal solutions derived from expected utility models [e.g., Wolf; Lapan, Moschini and Hanson; Hanson and Ladd]. This approach cannot be used to solve for optimal strategies since an expected utility model has no analytical solutions when options are in the choice set, and thus numerical search procedures are used to solve for global solutions (e.g., Hanson and Ladd). However, these solutions are usually expressed in fractions of contracts purchased or sold, and do not reflect the restriction implied by futures and options contracts of fixed sizes. This is especially important in analyzing models containing both futures and options, since options, with their selection of strike prices, can mitigate the effects of fixed futures contract sizes (Hauser and Andersen).

Here, numerical procedures are used to search for solutions of integer positions in the futures and option markets. The maximization procedure evaluates each possible combination of puts, calls, and futures contracts by numerically integrating the utility of return $R$ (8) achieved over a joint probability distribution of cash and futures prices as indicated in (9). In the base scenario, the producer agrees with the market's forecasts of mean and volatility of the ending price distribution. The parameters of the density function $L'(y, f^2)$ are changed in other scenarios in order to analyze the choice of marketing strategies as the producer's assessments differ from the market forecasts.
Utility Specification

Many empirical analyses have evaluated hedging strategies using a mean-variance (MV) framework. The MV framework is consistent with expected utility maximization when utility is quadratic or when the possible outcomes are normally distributed. These conditions are violated when options positions can be taken, although in many cases the MV may be a good approximation (Hanson and Ladd; Garcia, Adam, and Hauser). Additionally, since options positions can result in skewed outcome distributions, it may be important to examine a producer’s preference for skewness as well as the first and second moments (Cox and Rubinstein, p.318).

Using an actual utility function in (3), as opposed to an approximation or derivation such as the MV, makes use of the entire density function of returns, including the third and higher moments. The negative exponential utility function, \( U(R) = -\exp(-qR) \), where \( q \) is the Arrow-Pratt coefficient of absolute risk aversion, is used in this analysis.\(^3\)

Fitting the model with parameters appropriate for a hog producer, a value of \( q \) for a risk averse producer (\( q = 0.030 \)) is specified from a range suggested by Holt and Brandt for hog producers. To express differences among strategy outcomes in risk-adjusted money terms, certainty equivalent (CE) for the negative exponential utility function is used, where

\[
CE = \frac{-\ln(-EU(R))}{q}
\]

For each possible set of mean and volatility values for the true price distribution (48 in all, plus the market’s forecast), and for each set of forecasts (producer assessments) of the true price distribution, the value of forecast information is calculated as in (7) above. Only forecasts that are more accurate (less biased) than the market’s forecasts are considered, where more accurate forecasts are defined as those in which the forecast of at least one of the two parameters is more accurate than the market’s forecast and the forecast of the other one is at least as accurate as the market’s forecast. This results in 360 observations for value of information, of which one fourth have both mean and volatility forecasts greater than or equal to the market’s forecasts, one fourth have mean forecast greater than or equal to the futures price and volatility forecast less than the market’s implied volatility, one fourth have mean forecast less than the futures price and volatility forecast greater than or equal to the market’s implied volatility, and one fourth have both mean and volatility forecasts less than the market’s forecasts.

\(^3\)The use of the negative exponential function has been criticized because it assumes constant absolute risk aversion. The use of a decreasing absolute risk aversion utility function produced almost identical results to those reported later. The results of the negative exponential formulation are presented because of its ease of understanding and its frequent use in the literature.
Results

The results from the simulation are summarized by a regression which explains the change in value of information as a function of improved mean and volatility forecasts. A quadratic function with an interaction term between the improvements in mean and volatility is used:

\[
\%\Delta \text{Value} = \alpha + \beta_1 \%\Delta \text{Mean} + \beta_2 (\%\Delta \text{Mean})^2 + \gamma_1 \%\Delta \text{Vol.} + \gamma_2 (\%\Delta \text{Vol.})^2 + \delta(\%\Delta \text{Mean})(\%\Delta \text{Vol.}),
\]

where \(\%\Delta \text{Value}\) is the percent change in CE, \(\%\Delta \text{Mean}\) is the percent improvement in information about the mean of the period 2 price distribution, \((\%\Delta \text{Mean})^2\) is the percent mean improvement squared, \(\%\Delta \text{Vol.}\) is the percent improvement in information about the period 2 volatility, \((\%\Delta \text{Vol.})^2\) is the percent volatility improvement squared, and \((\%\Delta \text{Mean})(\%\Delta \text{Vol.})\) is the interaction term between percent mean improvement and percent volatility improvement.

The results of this regression are shown in Table 1. The significant variables are the linear and interaction terms. The signs indicate a positive relationship between value of improved information and the improved forecasts of the mean, volatility, and the interaction of the two. Hence, more accurate forecasts of the mean and volatility and their interaction contribute value.

These estimates indicate that the value of a more accurate forecast of the mean of the price distribution is about 10 times higher than the value of a more accurate forecast of the volatility. For every one percent improvement in the mean forecast, there is slightly more than a one percent improvement in the value of information. However, for a one percent improvement in the volatility forecast, there is only slightly more than a 1/10 percent improvement in the value of information. This result is consistent with the result of Chopra and Ziemba, who found in a portfolio context that in a mean-variance framework errors in mean estimation caused about 10 times as much loss as errors in volatility estimation. Also, Bullock and Hayes found that information about the mean of a price distribution improves access value of options more than does information about variance of the price distribution.

Additionally, the significant estimate of the interaction term indicates that the value of an improved forecast of one parameter is enhanced by an improvement in the forecast of the other parameter. For example, a 10% improvement in only the forecast of volatility increases the value of information about 1.1%, and a 10% improvement in only the forecast of mean increases the value of information by about 11%. However, a 10% improvement in the forecast of both mean and volatility increases the value of information by 12.9%.

Figure 1 illustrates this effect. The lowest line shows the percent improvement in value of information from improved information about the volatility, and the second line

\[4\text{From equation (7), } \%\Delta \text{Value} = (\text{CE}_p(X_2) - \text{CE}_p(X_1))/\text{CE}_p(X_1).\]
shows percent improvement from improved information about the mean. The top line shows the added improvement in value of information when information about both mean and volatility is improved by the same amount. The interaction effect may arise because improved information about the volatility of the price distribution provides an improved measure of the degree of confidence the producer may place in an improved forecast of the mean. For example, information that volatility is less than the market’s forecast increases the producer’s confidence in his assessment of price mean. This allows the producer to take riskier positions with higher returns, exploiting more fully the information about the mean.

Summary and Implications

This paper measures the value of more accurate forecasts of mean and volatility of hog prices to a hog producer making marketing decisions. The producer’s decisions are modeled in an expected utility framework in which marketing strategies consist of various combinations of futures contracts and put and call options. The producer chooses these strategies based on assessments about the distribution of cash and futures prices. Futures and options prices provide a market forecast of the mean and volatility of prices, and the value of more accurate information about price mean and volatility is measured using certainty equivalents.

The findings indicate that improved information about the mean of the price distribution is worth about ten times as much as improved information about the volatility of the price distribution to a risk averse producer. However, there is a significant interaction effect, with improved information about price volatility increasing the value of information about price mean.

These results suggest that relatively more effort should be devoted to forecasting price mean than to forecasting price volatility, since the value of a more accurate forecast of the mean is substantially higher than the value of a more accurate forecast of volatility. However, some effort should be devoted to forecasting volatility, since more information about volatility increases the value of information the hedger has about mean.
Table 1. Value of more accurate forecasts of mean and volatility of ending price distribution.

| Variable                                                                 | Estimate | Standard Error | t-value | Prob. > |t| |
|--------------------------------------------------------------------------|----------|----------------|---------|----------|-----------|
| percent improvement in mean forecast                                    | 1.065    | 0.162          | 6.579   | 0.000    |           |
| (percent improvement in mean forecast)$^2$                             | 0.336    | 1.740          | 0.193   | 0.847    |           |
| percent improvement in volatility forecast                              | 0.112    | 0.048          | 2.308   | 0.022    |           |
| (percent improvement in volatility forecast)$^2$                        | -0.028   | 0.155          | -0.178  | 0.859    |           |
| (percent improvement in mean forecast) x (percent improvement in volatility forecast) | 1.118    | 0.579          | 1.930   | 0.054    |           |
| constant                                                                | -0.003   | 0.003          | -0.802  | 0.423    |           |

Note: Dependent variable is percent improvement in value of information. Adjusted $R^2 = 0.61$, d.f. = 336

Figure 1. Percent Improvement in Value of Information Resulting from Improved Forecasts of Mean, Volatility, and Both Mean and Volatility.
References:


