Optimal Financial Leverage and the Determinants of Firm’s Hedging Policies under Price, Basis and Production Risk

by

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A new theoretical model of hedging is derived. Risk neutrality is assumed. The incentive to hedge is provided by progressive tax rates and bankruptcy laws. Optimal hedge ratios and the relationship with leverage, yield risk, basis risk, and financial risk is determined using alternative assumptions. An empirical example is provided to show changes in assumptions affect optimal hedge ratios for a wheat and stocker steer producer. Results show that hedging increases with increasing leverage and and increases at an even higher rate when the probability of bankruptcy is positive. The trade off between the cost and the tax-reducing benefits of hedging affects significantly the decision to hedge. The farmer wants to hedge to reduce tax payments, expected bankruptcy losses, interest rates and decrease the probability of bankruptcy. Even when the cost of hedging is a small portion of total cost, farmers may have no incentives to hedge when this cost is higher than the reduction in tax payments and the firm is in good financial position (i.e. low leverage ratios). For high levered firms, hedging reduces expected bankruptcy losses, and this effect may be considerably greater than the cost of hedging, making hedging very attractive. It is also shown that as basis risk increases, the optimal decision may well be not hedge at all.

Introduction

Empirical research usually finds optimal hedge ratios close to one (Ederington; Howard and D’Antonio; Kolb and Okunev; Mathews and Holthauser; Peck). Recently, Lapan and Moschini added basis and yield risk and found lower, but still high, optimal hedge ratios. The reality is that producers hedge much less. In a sample of 539 Kansas farmers, Schroeder and Goodwin found that, depending upon the crop, only 2% to 10% of the producers raising crops hedged. Tomek argues the hedge ratio is overestimated due to omission of important costs from the specification of farmers’ objective function (i.e.; yield risk, transaction costs). Lence finds that under realistic conditions (compare to minimum-variance hedge models) the optimal hedging strategy is simple not to hedge. Shapiro and Brorsen found that next to income stability (i.e., low income variability), the most important factor explaining the use of futures markets is the individual’s debt position. An appropriate model might be one in which the optimal debt position is determined within the model. The model should explicitly distinguish between a low-leveraged farmer who has little risk of bankruptcy, and may have no need to hedge, and a high-leveraged farmer who would hedge more because of his/her higher financial risk. This paper presents an optimal hedge model that will better explain what firms do in the real world.

Firms are normally assumed risk averse. However, empirical evidence shows that risk preferences are not significantly related to hedging (Shapiro and Brorsen). Schroeder and Goodwin found that risk preferences of crop producers did not influence forward pricing.

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Williams, Smith and Stulz, and Brorsen show that risk aversion is not necessary for firms to hedge. Rather than assuming risk aversion, this paper assumes that incentives exist to maximize firm's equity, allowing interest rates to vary according to the probability of bankruptcy. It is also shown that progressive tax rates are an important incentive to hedge. As Smith and Stulz argue, tax-reducing benefits of hedging become more significant as the function yielding the after-tax income becomes more concave. The model presented here will allow debt, and therefore leverage, to be determined endogenously. Tax liabilities, bankruptcy costs, cost of hedging, yield risk, cash price variability, basis risk and capital structure affect the firm's decision to hedge.

It has been widely accepted that output and price risk should be considered together when estimating optimal hedge ratios. Little has been done to incorporate financial risk into optimal hedge models. Schroeder and Goodwin found that leverage was positively related with forward pricing, and Harris and Baker’s survey indicates that hedging increases a farmer’s loan limits. Brorsen develops a theory where interest rates are a non linear function of initial wealth, debt, and the variability of ending wealth. Our model extends Brorsen’s model by explicitly incorporating the ability of hedging to reduce bankruptcy costs and tax liabilities. Also, the non linear interest rate function is estimated within the model by determining the expected bankruptcy losses to debt holders. This makes the model easier to estimate empirically as it is shown in this paper. This model will be of special use to analysts who are designing hedging strategies for firms and hope to produce recommendations that firms can actually use.

In this study, a new theoretical model of hedging is derived. Optimal hedge and leverage ratios and their relationship with yield risk, price variability, basis risk, and financial risk are determined using alternative assumptions about the value of the parameters. A simplified version of the model is estimated empirically for a wheat and stocker steer producer. The model is simulated using Monte Carlo integration and numerical derivatives, as opposed to comparative statics, are used to obtain the response of the key decision variables to variations in the parameters of the model. Results show that optimal hedge ratios are significantly higher for high levered firms. The model explains the empirical evidence of bank’s lending rates varying as much as five percentage points depending on the firm’s probability of bankruptcy. Similar to Lence’s findings, cost of hedging influences greatly the decision to hedge. Contrary to Lence, whose result is based on alternative investments that the agent may have, our result is based on the trade off that exists between the cost and the tax-reducing benefits of hedging.

The Model

Assume that because of unpredictable conditions (i.e., weather variability), the decision maker cannot predict output with certainty at the time of the production decision. The technology of the firm can then be represented by the stochastic production function:

\[ y_t = f(K_{t-1}, X_{t-1}, \varepsilon_t) \]
where \( y_t \) is an \( n \)-dimensional vector of production levels at time \( t \) (letting \( t = 1 \) be the harvest time and \( t-1 \) the time where decisions are made); \( f' \) is an increasing function of \( K_{t-1} \) and \( X_{t-1} \). Where \( X_{t-1} \) and \( K_{t-1} \) are respectively vectors of variable inputs and capital assets, and \( \epsilon_{it} \) is an \( n \times 1 \) vector of error terms.

Assume a competitive producer whose only available hedging instrument is a futures contract. Let \( F_{t-1} \) be an \( n \)-dimensional vector of the amounts hedged in the futures market. For each unit of \( F_{t-1} \), the firm makes \( (P_t' - P_t') \); the difference between the selling and the buying price of the futures contract. In the cash market, the firm makes \( (P_t' + b_t) \) dollars for each unit of output sold, where the basis \( (b_t) \) is the difference between the cash price \( (P_t') \) and the futures price at harvest \( (P_t') \). The firm's variable costs consist of the cost of inputs \( (P_t' X_{t-1}) \), the cost of hedging \( (h_{t-1} F_{t-1}) \), interests on debt \( (i_t D_t) \), and depreciation \( (\alpha K_{t-1}) \), where \( P_t'X_{t-1} \) is a vector of input prices, \( h_t \) is the cost of hedging, \( i_t \) is the interest rate, \( D_t \) is total liabilities, and \( \alpha \) is a (constant) rate of depreciation. The firm's profit at time \( t \) \( (\pi_t) \) can then be represented by

\[
\pi_t = y_t' (P_t' + b_t) + F_{t-1}' (P_t' - P_t') - P_t' X_{t-1} - h_t F_{t-1} - i_t D_t - \alpha K_{t-1}
\]

Note that because all payments to debt claims are assumed to be tax deductible, the firm's profit \( (\pi_t) \) is the same as taxable income and interests are therefore subtracted in (2). Total liabilities \( (D_t) \) can be determined by computing the capital needed to finance inputs, hedging cost, and investment, after beginning equity has been used \( (w_{t-1}) \), and adding the liabilities from the previous period:

\[
D_t = P_t' X_{t-1} + h_t F_{t-1} + (K_t - K_{t-1}) - W_{t-1} + (D_{t-1} - A_{t-1})
\]

where \( (K_t - K_{t-1}) \) is capital investment, \( (D_t - A_{t-1}) \) is debt brought from previous period and \( A_{t-1} \) account for any amortization made to the principal at \( t-1 \).

Denote \( \tau(\pi_t) \) as the firm's income tax rate, which is an increasing function of profits \( (\pi_t) \); therefore, net income \( (NI_t) \) or after tax income is

\[
NI_t = \begin{cases} 
[1 - \tau(\pi_t)] \pi_t; & \pi_t > 0 \\
\pi_t; & \pi_t < 0 \\
-W_{t-1}; & \pi_t < -W_{t-1}
\end{cases}
\]

---

1 This is an approximation since this function does not show the progressivity of taxes as suggested by the U.S. tax code.
indicating that taxes are zero whenever profits are negative, and net income can be negative whenever profits are negative, but its absolute value cannot be less than beginning equity (i.e., the firm cannot lose more capital than it has available).

The firm’s net worth would be net income plus beginning equity:

$$W_t = NI_t + W_{t-1}.$$  

Since the firm cannot lose more capital than it has available, ending wealth ($W_t$) must be positive and net income is bounded to be greater than the negative of beginning equity ($NI_t > -W_{t-1}$). Note that the firm will be bankrupt when profits are negative and less than the negative of initial wealth ($\pi_t < -W_{t-1}$). Also note that when profits are equal to the negative of initial wealth ($\pi_t = -W_{t-1}$), net income equals initial wealth ($NI_t = -W_{t-1}$) (3) and the firm will be at the boundary of going bankrupt (i.e., $W_t = 0$) (4). When ($W_t = 0$) and the amount ($W_{t-1} - \pi_t$) is positive, the firm is bankrupt and the losses to debt holders ($LS_t$) are:

$$LS_t = C(D_j) + W_{t-1} - \pi_t; \quad \pi_t < -W_{t-1},$$

where $C(D_j)$, the bankruptcy fee, is an increasing function of the debt level.

Let us now further define some of the variables introduced so far in the model. The decision maker knows at time $t-1$ the vector of futures prices ($P_{t-1}^f$), the price vector of inputs ($P_{t-1}^c$), and the cost of selling in the futures market ($h_t$). However, because of production lags the farmer does not know with certainty the vector of output prices in the cash market ($P_t^c$), or the futures price at harvest ($P_t^f$).

The random variables, ($P_t^f$) and ($P_t^c$), can be defined as following a random walk

$$P_t^c = P_{t-1}^c + \epsilon_{2t}$$

$$P_t^f = P_{t-1}^f + \epsilon_{3t}$$

where $\epsilon_{2t}$ and $\epsilon_{3t}$ are vectors of mean zero errors. The deterministic part of the basis ($h_t$) equals ($P_{t-1}^c - P_{t-1}^f$) and ($\epsilon_{3t} - \epsilon_{2t}$) denotes the random part of the basis.

Farmers face risk (i.e., yield risk, basis risk, price risk); therefore, bankruptcy is possible since the bankruptcy condition ($\pi_t < -W_{t-1}$) defined in (6) can occur with positive probability. Assuming banks are risk neutral, lenders will charge the firm a premium which would be a function of the expected bankruptcy losses. Hence a firm with a higher probability of bankruptcy would have a higher expected rate of interest on its debt than a firm with lower financial risk. In practice, interest rates can vary by as much as five percentage points. It is
assumed here that banks have enough information about the firm to calculate the expected bankruptcy losses and charge a premium to firms are likely to go bankrupt. The firm’s expected interest rate on debt (\(i_t\)) will then equal the prime interest rate (\(r^*\)) plus a premium (\(PR_t\))

\[
(8) \quad i_t = r^* + PR_t
\]

where the prime interest rate (\(r^*\)) is assumed constant over time.

Given the bankruptcy losses (\(LS_t\)) defined in (6), the premium (\(PR_t\)) can be defined as the expected ratio of the losses to total liabilities (\(LS_t/D_t\)):

\[
(9) \quad PR_t = \int \int \int \left[ \pi_t < -W_{t-1} \right] \frac{LS_t}{D_t} f(\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}) d\epsilon_{1t} d\epsilon_{2t} d\epsilon_{3t}
\]

where \(I[.]\) is an indicator function that takes the value of one if the firm goes bankrupt and zero otherwise. \(f(\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t})\) is the probability density function of the error terms defined in equations (1) and (8). The premium (\(PR_t\)) at time \(t\) would be zero if the probability of the firm going bankrupt is zero so that the ratio of losses (\(LS_t\)) to the debt level (\(D_t\)) equals zero.

Assume that the hedger aims to maximize the expected net present worth (\(W_t\)), given the expected interest rate (8) and the non-bankruptcy condition (\(W_t > 0\)), yielding the objective function:

\[
(10) \quad \text{Max } E[W_t] = W_0 + \sum_{i=1}^n \int \int \int \beta^t W_t I[W_t > 0] f(\epsilon_{1t}, \epsilon_{2t}, \epsilon_{3t}) d\epsilon_{1t} d\epsilon_{2t} d\epsilon_{3t}
\]

where \(\beta^t\) is the discount factor (0 > \(\beta^t\) < 1), and \(W_0\) is the exogenous initial wealth. In (10), when net worth (\(\pi_t < -W_{t-1}\)) is positive, \(W_t\) will equal zero, indicating that equity holders get nothing if the firm enters bankruptcy. At \(t = 1\), the firm has to choose the input levels (\(X_{1,1}\)), how much to hedge (\(F_{1,1}\)), and the level of capital (\(K_t\)). The levels of output and debt are determined by equations (1), and (3).

**Empirical Model**

Because of the complexity of the analytical model, we were unable to derive comparative statics results. Therefore, the effects of various factors are determined numerically for a specific example. Simulations are performed for a wheat and stocker steer producer. A simplified version of the model is simulated here by assuming a constant returns to scale production process and a one-period production function. Also, government programs for wheat are not included. The producer has made the decision to produce 1000 acres of wheat, and graze 296 steers on the winter wheat pasture. The wheat is planted in September and harvested June, and the farmer buys steer calves in October and sales them in March. The only decision left to make is how much of the expected output of wheat and steers should be hedged in the futures
market. The basic data for simulation is set forth in Table 1. Assume that wheat yields and the safe weight of the animals (\( e_t \)) follow a beta distribution with mean and standard deviations shown in Table 1. Cash and futures prices follow a multivariate normal with a correlation coefficient of 0.9. The amount of capital borrowed will be dependent on the cost of hedging which depends on the amount hedged in the futures market and on beginning equity \( (W_t) \).

Monte Carlo integration was used to integrate functions (9) and (10). The beta distributions were generated by first drawing two independent samples from a Gamma distribution using the Phillips generator (Shannon). These two samples are then combined to get beta random numbers (Shannon). Cash and futures prices were generated assuming that futures markets are efficient \( (E(P_t^f) = E(P_t^c)) \). Samples of size 1000 were generated using antithetic variates as variance reduction technique (Shannon). The model was solved by means of the non linear algorithm in GAMS, which uses analytical derivatives. The problem encountered was that the tax and the bankruptcy loss function (6) are non differentiable at zero. To deal with this, the following transformation was made:

\[
\sqrt{(x^2 + x)} \over 2
\]

where \( x \) can be either taxes of bankruptcy losses. Transformation (11) makes variable \( x \) differentiable and greater than zero. Note that the bankruptcy loss function (6) is both non differentiable and discontinuous. The discontinuity was approximated by making the bankruptcy fee (\( C(D_t) \)) a function of the losses; therefore, (6) becomes:

\[
LS_t = (W_{t-1} - \pi_t) (1 + 0.3) ; \; \pi_t < - W_{t-1}
\]

where 0.3 is the bankruptcy fee rate (Table 1).

Given the complexity of the objective function (12), numerical derivatives were employed to obtain the response of the key decision variables to variations in the parameters of the model.

**Discussion of Results**

**Leverage**

The model shows that hedging increases with increasing leverage ratios (Figure 1). When the firm shows positive probability of bankruptcy (Figure 2), hedging increases significantly more. Note that the results hold under the assumption of risk-neutrality whereas in Turvey and Baker, similar response is obtained for risk-averse producers. For low levered firms (0.1 to 0.5), increases in hedging reduces tax payments for farmers. As leverage increases, the incremental tax advantage of borrowing declines and the interest tax shield becomes less certain, making hedging less attractive (Myers). In accordance with Smith and Stulz's model, once the firm reaches levels of bankruptcy, hedging can significantly reduce the probability of bankruptcy. Hedging reduces the variability of profits and the expected corporate tax liability which in turn increases the expected ending wealth (Smith and Stulz).
The model yields relatively low hedge ratios for steers from less than 0.1 to about 0.6. For a sample of Kansas farmers with an average leverage ratio of 0.4, Schroeder and Goodwin find that the average percent of cattle sold using futures hedge was 25.4%. Our results predict closely their findings. On the other hand, our model predicts relatively high hedge ratios for wheat (0.4 to .85), while Schroeder and Goodwin find the sample average of wheat hedge ratios was 0.22. One possible explanation of the divergence is that government programs have not yet been incorporated in this model. Also, wheat is relatively riskier than cattle because of the longer time between planting and harvest while steers are only owned a short time.

**Interest Rates**

As Collins and Karp argue, increases in leverage also increases the probability of a disaster which cause increased risk of loss for the lender, and the cost of borrowing also increases with leverage. This relationship is shown in figure 3. When the probability of bankruptcy is non-zero, the interest rate rises above the riskless interest rate. For this simulation example, the firm shows positive probability of bankruptcy at relatively high leverage ratios (above .85), where the cost of borrowing begins to rise.

**Cost of Hedging**

Traditional optimal hedging models introduced by Johnson and Stein and variations of their approach (i.e., Myers and Thompson) assume away the cost of hedging. In this paper, it is shown that the cost of hedging influences greatly the decision of whether or not to hedge and how much to hedge (Figure 4 and 5). Hedging cost can cause a cattle producer not to hedge (figure 5) and the optimal hedge ratios for wheat can vary from a high of 0.80 to 0.40 when hedging cost goes from 1 to 3 cents per pound, respectively. Clearly, transaction costs of hedging may well exceed the tax-reducing benefits of hedging, causing hedge ratios to decrease.

**Basis Risk**

There is a significant response of hedging to changes in basis variance (figure 6 and 7). As basis variance increases, the correlation between futures and cash prices goes down, and producers will have less incentive to hedge. Wheat optimal hedge ratios fall from 0.75 to 0.45 when the correlation of cash and futures prices decreases from 0.9 to 0.78. For steers, the hedge ratio can be close to zero if the basis variance increase up to 0.12 (i.e., correlation coefficient of 0.78). The model also shows that when basis variance of one commodity increases, hedging of other commodities can also decrease. This cross effect is, however, considerably lower than the direct effect of basis risk.

**The Role of Taxes**

In this model, taxes play a very important role in the decision to hedge, especially when the firm is not close to bankruptcy. A 30% tax, down from a 40% tax, can make the steer hedge ratio go to zero (figure 8). A tax rate of 20%, on the other hand, can reduce the wheat hedge ratio down to 0.3. This result that with this model one should consider the actual value of hedge ratios with caution. The real contribution of the paper is to show how hedging is affected by leverage, probability of bankruptcy, cost of hedging and basis risk. A better way to model the tax function to show a progressive tax structure, as the U.S. tax code suggests, is necessary. This is important since the tax-reducing benefits of hedging may increase if the function that yields after-tax income (3) becomes more concave (Smith and Stulz).
Conclusions

This paper has shown that risk-averse preferences are not necessary for farmers to hedge. Further, risk aversion is not necessary to explain why interest rates charged by banks differ from firm to firm. Tax rates and bankruptcy costs provide the concavity of the objective function necessary to motivate firms to hedge. This paper provides an explanation of why some firms do not hedge or hedge more than others. Leverage, expected bankruptcy losses, basis risk, tax liabilities, and the cost of hedging play an important role in deciding whether to hedge and how much to hedge. Hedging increases significantly with increasing leverage and more significantly when the probability of bankruptcy is positive. The trade off between the cost and the tax-reducing benefits of hedging affects significantly the decision to hedge. The farmer wants to hedge to reduce tax payments, interest rates and to decrease expected bankruptcy losses.

Even when the cost of hedging is a small portion of total costs, farmers may have no incentives to hedge when hedging cost is higher than the reduction in tax payments and the firm is in good financial position (i.e. low leverage ratios). For high-levered firms, hedging reduces expected bankruptcy losses, and this effect may be considerably greater than the cost of hedging, making hedging very attractive. It is also shown that as basis risk increases, hedging becomes less and less effective in reducing profit variability, and the optimal decision may well be not hedge at all. Through the reduction of the firm’s profit variability, hedging reduces expected tax liabilities, which in turn increases expected ending wealth if the cost of hedging is not too high. As the variability of profits decreases, the expected bankruptcy losses decrease, cutting down interest rates charged by banks. The model explains empirical evidence that borrowing rates increase at an increasing rate as leverage increases.

References


### Table 1. Wheat for Grain, Owned Harvest Equipment, Budget Per Acre and Stocker Steers on Wheat Pasture Cost/Returns Per Head

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Value</th>
<th>Standard Deviation</th>
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<tr>
<td>Wheat:</td>
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<tr>
<td>Variable Cost</td>
<td>$/acre</td>
<td>78.32</td>
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<tr>
<td>Capital investment</td>
<td>$/acre</td>
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<td>Yield</td>
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<td>5.00b</td>
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<td>Futures price at planting</td>
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<tr>
<td>Futures price at harvest</td>
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<td>Cost of hedging</td>
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<td>Steers:</td>
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<tr>
<td>Variable Cost</td>
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<td>Capital investment</td>
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<td>Steers calf weight</td>
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<td>Sale weight</td>
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<td>Price of steers calves</td>
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<td>Cash price</td>
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<td>Cost of hedging</td>
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<td>Bankruptcy fee</td>
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<td>30.00</td>
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</tr>
<tr>
<td>Interest rate</td>
<td>%</td>
<td>8.50</td>
<td>-</td>
</tr>
</tbody>
</table>

*The costs for production were taken from the OSU Enterprise Budgets developed by Oklahoma State University, Department of Agricultural Economics.

b Source: Koontz and Trapp

c Source: Schroeder and Goodwin
Figure 1. Optimal Hedge Ratios and Leverage for a Wheat and Stocker Steer Producer

Figure 2. The Probability of Bankruptcy as Leverage Increases
Figure 3. Expected Interest Rates as Leverage Increases

Figure 4. Optimal Hedge Ratios and The Cost of Hedging Wheat
Figure 5. Optimal Hedge Ratios and the Cost of Hedging Steers

Figure 6. Optimal Hedge Ratios for Wheat and Steers as the Basis Variance for Wheat Increases
Figure 7. Optimal Hedging Ratios for Wheat and Steers as the Basis Variance for Steers Increases

Figure 8. Optimal Hedging Ratios and the Tax Rate