Impacts of Government Risk Management Policies on Hedging in Futures and Options: LPM2 Hedge Model vs. EU Hedge Model

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Impacts of government risk management policies on hedging in futures and options: LPM$_2$ hedge model vs. EU hedge model

The main objective of this study is to compare the impacts of government payments and crop insurance policies on the use of futures and options measured from a downside risk hedge model with the impacts analyzed by the expected utility (EU) hedge model. Understanding the effects of government-provided risk management tools on the private market risk management tools, such as futures and options, provides value to both crop farmers and policy makers. Comparison of the impacts from the two hedge models shows that crop farmers will hedge less in futures under the LPM$_2$ model than under the EU hedge model. This finding indicates that model misspecification is another reason for the phenomenon that farmers actually hedge less in futures than predicted by the EU model. From the perspective of exploring new research techniques, this study applied two relatively new simulation concepts, copula simulation and conditional kernel density approach, to make the simulation assumptions less restrictive and more consistent with observations. The copula simulation applied in this study allows yield and price to have more flexible joint distribution functions than multivariate normal; the conditional kernel density approach used in farm yield simulation enables the variance of farm yield varies with county yield rather than being constant.

**Keywords**: Down-side Risk, LPM$_2$ Hedge Model, Government Payments, Crop Insurance Policies, Copula Simulation, Conditional Kernel Density

**MOTIVATION**

The prevalence of government payments and government-initiated crop insurance policies has prompted the question, “how would government-provided risk management tools affect the use of private market risk management tools, such as futures and/or options contracts, by crop farmers?” Investigating such effects provides value to all potential participants, both public and private. Firstly, acknowledging that the availability of government-supported risk management tools will change the optimal hedging in futures and options contracts will benefit crop farmers in that they can adjust their risk management portfolio accordingly. Secondly, examination of such impacts could provide policy makers with insights into the question, “to what extent are the income risks faced by U.S. crop farmers managed under the interaction of government and private market risk management tools?”

Previous studies have approached this question by assuming that the objective of the crop producer is to manage risk by maximizing the expected utility of a portfolio, which includes government payments, crop insurance, futures and/or options (Poitras, 1993; Hanson et al. 1999; Coble et al. 2000, 2004; Mahul 2003; Wang et al. 2004). However, concerns with the expected utility (EU) hedge model enter from two perspectives. First, an expected utility function must be assumed in the analysis, which may result in misleading findings and lack of generality (Sakong et al., 1993). Second, EU maximization does not target the control of downside risk. If we assume that government payments and crop insurance reflect demands by farmers with respect to risk management, then a downside risk hedge model might be a more appropriate model than the EU hedge model, because the ultimate purpose of the government payments and federal crop
insurance plans is to protect crop farmers from receiving low farm incomes, as indicated by the often-cited “Safety net” concept in U.S. farm policy files.

The main objective of this study is to compare the impacts of government payments and crop insurance policies on the use of futures and options measured from a downside risk hedge model with the impacts analyzed by the EU hedge model. In particular, the downside risk hedge model applied is the second-order lower partial moment hedge model ($LPM_2$), which minimizes the expected value of the squared shortfall from the target payoff. The hypothesis is that a crop farmer will hedge less in futures contracts under the $LPM_2$ model than under the EU hedge model. If the results confirm this conjecture, then misspecification of the hedging objective would be at least partial explanation for the observation that farmers actually hedge less in futures than predicted by the EU model.

METHODOLOGY

The lower partial moment criterion has thus far had its primary applications in the finance literature. Because the $LPM_2$ hedge model is proposed to investigate the effects of government programs on the optimal hedge in commodity futures and options contracts rather than the expected utility model, we present an analytical and empirical comparison between $LPM_2$ hedge ratios and the expected utility (EU) hedge ratios. This study applies the copula method to simulate the dependence between crop prices and yields so that prices and yields can have a non-normal univariate distribution and no explicit assumption on their joint distribution need to be made. We also employ the cumulative conditional kernel density approach to simulate farm yields so that the individual, or representative, farm yield can vary with the levels of county yields, consistent with observed patterns between historical farm yields and county yields. Effects of government direct payments (DP), loan deficiency payments (LDP) and counter cyclical payments, as well as the effects of two typical crop insurance policies, actual production history (APH) and crop revenue coverage (CRC), are compared across the two models.

**Downside Risk Measures, $LPM$, and Stochastic Dominance**

Using $LPM$ as downside risk measures originated in the investment literature. Roy (1952) proposed to apply the safety-first rule to make investment decisions under uncertainty. Roy’s safety-first rule determines the optimal investment portfolio by minimizing the probability of the investment return falling below some target threshold return while maintaining a specified expected return. Thus, Roy’s portfolio selection criterion reflects a downside risk measure which defines risk as the shortfall relative to the threshold return. This risk measure is in contrast with the traditional risk measure, variance, which counts both the upside potential and the downside misfortune from the expected return as risk.

Fishburn (1977) suggested to measure risk with an $\alpha - t$ model in the form

\[ Risk = \int_{t}^{\infty} (t - x)^{\alpha} dF(x), \] 

where $\alpha > 0$ and $F(x)$ is the cumulative distribution function. Fishburn’s risk measure is motivated by the observation that decision makers in the investment context usually associate risk with failure to attain at least a target return. Bawa (1978) generalized Roy’s downside risk measure by constructing a set of downside risk measures, called lower partial moments, in the form
\[
LPM_n = \int_{-\infty}^{\pi} (\pi - \bar{\pi})^n dF(\bar{\pi})
\]  
(1),

where \( n \geq 0 \). \( LPM_0 \) is equivalent to Roy’s down-side risk measure. When \( n \) is a positive integer, \( LPM_n \) represents the probability-weighted \( n^{th} \)-power of the shortfall below a target payoff \( \pi \). Thus, Bawa’s LPM risk measures are equivalent to risk measured by Fishburn’s \( \alpha - t \) model.

The theoretical justification of using the LPM criterion to determine an optimal investment portfolio resides in the relationship between LPM risk measures and stochastic dominance. Stochastic dominance rules rank the alternative investment portfolios based on the payoff distribution of the portfolios. The \( n^{th} \)-order stochastic dominance is defined as follows. Given two portfolios \( X \) and \( Y \), and their respective cumulative payoff distribution \( F \) and \( G \), if \( F^{(n)}(t) \leq G^{(n)}(t) \) for \( t \in R \), then \( X \) dominates \( Y \) by the \( n^{th} \)-order stochastic dominance.

Here, \( F^{(n)}(t) = \int_{-\infty}^{t} F^{(n-1)}(u) du \), \( F^{(1)} \equiv F(t) \), \( n \geq 2 \), and \( F(t) \) is the cumulative distribution function (Levy, 1998).

In particular, the first-, second-, and third-order stochastic dominance are defined as follows (Levy, 1998).

Definition 1: If \( F(t) \leq G(t) \) for all values of \( t \) and if at least at some \( t^* \) that \( F(t^*) < G(t^*) \), then portfolio \( X \) dominates portfolio \( Y \) by first-order stochastic dominance (X FSD Y).

Definition 2: If \( F^{(2)}(t) \leq G^{(2)}(t) \), (or \( \int_{-\infty}^{t} F(u) du \leq \int_{-\infty}^{t} G(u) du \) for all \( t \) and the inequality holds at least at some \( t^* \), then portfolio \( X \) dominates portfolio \( Y \) by second-order stochastic dominance (X SSD Y).

Definition 3: If \( F^{(3)}(t) \leq G^{(3)}(t) \), (or \( \int_{-\infty}^{t} \int_{-\infty}^{u} F(u) du dz \leq \int_{-\infty}^{t} \int_{-\infty}^{u} G(u) du dz \) for all \( t \) and the inequality holds at least at some \( t^* \), and \( \int_{-\infty}^{t} F(u) du \leq \int_{-\infty}^{t} G(u) du \), where \( b \) is the upper bound of \( t \), then portfolio \( X \) dominates portfolio \( Y \) by third-order stochastic dominance (X TSD Y). \(^2\)

The \( n^{th} \)-order stochastic dominance is consistent with expected utility maximization for all utility functions satisfying \((-1)^k U^{(k)}(0) \leq 0 \) (\( k = 1, 2, ..., n \), \( U^{(k)} \) is the \( k^{th} \) derivative of function \( U \). (Yamai and Yoshida, 2002; See pp.116-117 of Levy, 1998 for the proof). Because the utility functions discussed in the economics and finance literature are usually assumed to satisfy non-satiation (\( U' > 0 \)), risk-aversion (\( U'' < 0 \)) and decreasing absolute risk aversion (a necessary condition is \( U'' > 0 \)), the relationship between FSD, SSD, TSD and the expected utility with such properties are given by the following theorems.

\(^2\) That \( \int_{-\infty}^{t} F(u) du \leq \int_{-\infty}^{t} G(u) du \) is equivalent to \( E_F(X) \geq E_G(Y) \) can be proved by applying integration by parts (Levy, 1998).
Theorem 1: If and only if X FSD Y, then \( E_U(X) \geq E_U(Y) \) for all the utility functions with \( U' > 0 \), and strict inequality exists for at least some \( U_0 \) with \( U'_0 > 0 \). (See pp.48-49 of Levy, 1998, for the proof.)

Theorem 2: If and only if X SSD Y, then \( E_U(X) \geq E_U(Y) \) for all the utility functions with \( U' > 0 \) and \( U'' < 0 \), and strict inequality exists at least for some \( U_0 \) with \( U'_0 > 0 \) and \( U''_0 < 0 \). (See pp.69-72 of Levy, 1998, for the proof.)

Theorem 3: If and only if X TSD Y, then \( E_U(X) \geq E_U(Y) \) for all the utility functions with \( U' > 0 \), \( U'' < 0 \) and \( U''' > 0 \) and at least for some \( U_0 \). (See pp.92-96 of Levy, 1998, for the proof.)

Both Fishburn (1977) and Bawa (1978) noted that LPM\( n \) criterion are consistent with \((n+1)^{th}\) order stochastic dominance rule for \( n = 0, 1, 2 \). That is:

- If X FSD Y, then \( LPM_0(X) \leq LPM_0(Y) \) for all target payoff; and
- If X SSD Y, then \( LPM_1(X) \leq LPM_1(Y) \) for all target payoffs; and
- If X TSD Y, then \( LPM_2(X) \leq LPM_2(Y) \) for all target payoffs.

By means of integrating by parts, Ingersoll (1987) proved \( F^{(n+1)}(t) = \frac{1}{n!} LPM_{n,t} \). Thus, the LPM\( n \) criterion is consistent with \((n+1)^{th}\) order stochastic dominance. Because \((n+1)^{th}\) order stochastic dominance and the EU criterion result in the same ranking for all the utility functions with \((-1)^k U^{(k)} \leq 0 \) (\( k = 1, 2, \ldots, n \)), LPM\( n \) criterion is also consistent with the EU criterion for all the utility functions \( U \) satisfying \((-1)^k U^{(k)} \leq 0 \) (\( k = 1, 2, \ldots, n \)). In particular, LPM\( 2 \) is consistent with the EU criterion when the EU criterion yields the same ranking for all the utility functions with \( U' > 0 \), \( U'' < 0 \) and \( U''' > 0 \), of which the usually desired decreasing absolute risk aversion utility functions are members (Harlow and Rao, 1989). Moreover, LPM\( 2 \) is equal to the semivariance when the target payoff in LPM\( 2 \) is set at the expected payoff, and thus LPM\( 2 \) is the exact downside risk measure comparable to variance, which treats both upside and downside deviations from the expected payoff as risk.

If the distribution of the portfolio payoffs is symmetric, then LPM\( 2 \) risk measure would be equivalent to variance and result in the same ordering of risky portfolios (Eftekhari, 1998). However, for an asymmetric portfolio payoff distribution, a downside risk measure such as LPM\( 2 \) is more intuitive than variance.

Relationship between LPM\( 2 \) Hedge Ratios and EU Hedge Ratios

When the target payoff is set to be the mean payoff, the optimal LPM\( 2 \) hedge ratios tend to increase the right skewness of the payoff distribution of the hedge portfolio. The reason is that the optimal LPM\( 2 \) hedge ratios are solutions that minimize the lower part of the expected variation relative to the mean.

On the other hand, how EU hedge ratios affect the payoff distribution of the hedge portfolio depends on many conditions, such as the presence of production risk and unbiased
futures prices\(^3\). Under the assumptions of no production risk, unbiased futures prices, and linearity of the relationship of cash prices with futures prices, the optimal hedge ratios from the EU hedge model in such conditions are the same as the optimal hedge ratios under a minimum variance hedge model (Lence, 1995; Lien and Tse, 2002). By examining the Taylor expansion of the EU function at the mean payoff of the portfolio, one important implication can be drawn: if the utility function is continuous, risk averse, and has at least up to third-order derivatives, then optimal hedge ratios that maximize EU tend to limit the variance of the portfolio payoff but favor the right skewness of the payoff distribution. Thus, optimal hedge ratios from an EU model would be a balance between the smaller variance and the higher right skewness (Lapan et al. 1991):

\[
EU(\pi) = \int u'(\mu_n + \delta) f(\delta) d\delta \\
= u'(\mu_n) + \frac{1}{2} u''(\mu_n)E(\pi - \mu_n)^2 + \frac{1}{3!} u'''(\mu_n)E(\pi - \mu_n)^3 + \ldots + \frac{1}{(n-1)!} u^{(n-1)}(\mu_n)E(\pi - \mu_n)^n \\
+ \frac{1}{n!} E[u^{(n)}(\pi + \epsilon \delta) \delta^n],
\]

where \( 0 < \epsilon < 1 \) and \( \mu_n \) is the mean of income \( \pi \).

Although both the LPM\(_2\) hedge and the EU hedge have preference for right skewness of the payoff distribution of the hedge portfolio, the analysis that the EU model favors smaller variance whereas the LPM\(_2\) hedge minimizes semivariance may indicate that the crop farmer will likely hedge less in futures contracts but more in options under the LPM\(_2\) model than under the EU hedge model.

The next section discusses and explains the assumptions and components to build the LPM\(_2\) hedge model for several portfolios that include futures, put options, crop insurance and/or government payments. The second part clarifies the process to generate the desired data for numerical optimization. Because the LPM\(_2\) hedge model proposed by this study has not been used previously to investigate the planting-time hedge decision by crop farmers, we will concentrate on establishing the LPM\(_2\) hedge model. Compared to the LPM\(_2\) hedge model, an EU hedge model requires the same portfolio and probability information but a different objective function. Therefore, once the LPM\(_2\) hedge model is set up and solved, optimal hedge ratios for the EU hedge model can be easily obtained as an extension.

**LPM\(_2\) Hedge Model for the Representative Crop Farmer**

Given the availability of Georgia’s farm-level yield data, the hedging decisions of a representative cotton farmer in Colquitt county of Georgia and of a representative soybean farmer in Bulloch county of Georgia will be investigated. Colquitt and Bulloch counties are selected because of their high acreage devoted to cotton/soybean production\(^4\) and their larger acreage.

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\(^3\) Lapan et al. (1991), Sakong et al. (1993), Mochini and Lapan (1995) defined that the futures price is unbiased if the expectation of the end-of-period futures price equals the current price of a futures contract.

\(^4\) In 2005, Bullock had the largest number of harvested soybean acres in Georgia; Colquitt had the second largest number of harvested cotton acres (GA Statistics System in 2005-2006 Georgia County Guide).
numbers of farmers with more than six years of continuous records during the period of 1991-
2000.

The representative farmer in a county is synthesized in such a way that for any year, the
mean of the yield distribution equals the county yield, but the actual yield could be any actual
farm-level yield realized in the county. The dispersions of farm-level yield around county yield
vary from year to year, indicating that the representative farm defined above should have
variance conditional on the county yield.

Other assumptions made for the representative producer are as follows. First, the crop
producer is assumed to make a one-time hedge through the crop year. That is, the crop farmer
takes hedge positions in futures and options at planting time, and later offsets his positions in
futures and/or options at harvest. This assumption rules out the scenario that the crop farmer sets
hedge positions at multiple time points from planting to harvest. Specifically, both the cotton and
soybean representative farmers are assumed to take on hedge position on the last trading day in
March, 2006. The cotton farmer will lift the hedge in November 2006, while the soybean farmer
will close the hedge in mid-October.

Second, the representative farmer is assumed to have a portfolio composed of income
from selling crops at spot market as well as from four risk management instruments – futures,
options on futures, federal crop insurance and government payments. To simplify the analysis
regarding government payments, the representative farmer is assumed to produce a single
commodity for which he is qualified to receive government payments. Third, the representative
farmer makes his/her hedging decision at planting time by utilizing information available at that
time. Such information includes parameters determined by the policy makers regarding the
federal crop insurance and government payments and the estimates of the yield and price
distribution at harvest. Fourth, the hedge decision for every acre of the crop planted is assumed
to be independent of total acres planted. That is, the hedge portfolio based on one acre of the
planted crop can be analyzed for simplicity.

Modifications to the Existing LPM₂ Hedge Model

The hedging problems examined so far by the LPM₂ hedging literature have considered
price uncertainty as the only source of risk. But for crop farmers to make hedging decisions at
planting time to protect their harvest income, unknown harvest yield is another factor that
contributes to the uncertainty of their hedge portfolio. Therefore, the existing LPM₂ hedge model
must be modified to include both price and production variables.

The presence of production uncertainty generally calls for the use of options together
with futures contracts (Sakong, 1993). Thus, put options will be included in the LPM₂ hedge
model in this study, and the optimal hedge ratio in options will be determined simultaneously
with the optimal hedge ratio in futures contracts.

Hedge Portfolio in the Base Scenario

When there are no insurance plans or government payments available to the
representative farmer, that farmer is assumed to use only futures and options to manage risk. The
net value of the hedge portfolio at harvest is
\[
\pi = py + h_x (f_0 - f_i)Ey + h_z [\max(f_i - k, 0) - v_z]Ey - C(Ey). \tag{3}
\]

Here, \(py\) is the revenue from selling the crop for cash at harvest, \(p\) is the cash price at harvest and \(y\) is the representative farm yield. Production cost \(C\) is assumed to be determined by the expected farm yield \(Ey\), and therefore is known at planting time. The production costs of per acre crop planted in 2006 were decided based on the cost estimated by Cooperative Extension at the University of Georgia\(^5\). The terms \(h_x\) and \(h_z\) are the hedge ratios for futures and options, respectively, measured as the ratios of yields hedged with futures contracts (or options contracts) to the expected farm yields. Positive (negative) \(h_x\) means to sell (buy) futures contracts. Positive (negative) \(h_z\) means to buy (sell) put options contracts. For example, if \(h_x = 0.5\), the interpretation should be that the farmer hedge 50% of the expected yield on futures market with futures contracts. Conversely, \(h_x = -0.5\) means that the farmer speculates in the futures market with the crop amount equal to 50% of his expected production.

An implicit assumption to such a definition of hedge ratios is that the farmer can purchase any fraction of a standard futures contract or options contract. \(f_0\) and \(f_i\) are the prices of the close-to-harvest futures contract at planting and at harvest, respectively. For cotton, \(f_0\) and \(f_i\) are the prices of the December contract, while for soybeans, they are prices of the November contract. \(k\) is the strike price of the put options, and \(v_z\) is the premium that the farmer pays to have the put options. Harvest cash price \(p\), harvest farm-level yield \(y\) and futures price at harvest \(f_i\) are not known at planting and will be simulated based on historical data. The assumption of unbiased futures and options prices will be imposed by simulating \(f_i\) in such a way that \(Ef_i = f_0\) and by setting option premium \(v_z = E(\max(f_i - k, 0))\).

**Hedge Portfolio Including APH or CRC Insurance Plan**

To investigate the impacts of federal crop insurance plans on the use of futures and options, insurance is added to the hedge portfolio. The two most-used crop insurance plans, yield insurance plan APH and revenue insurance plan CRC, are considered as the alternative insurance policies that the representative farmer will choose from. To focus on the effects of crop insurance on the demand for hedging in futures and options contracts, the representative farmer is not given the choice to determine the coverage level and price election in the insurance plans. However, sensitivity tests of coverage level of the APH or CRC insurance plan on the demand for futures and options are performed, and the results are discussed in the next section.

The value of the hedge portfolio including APH plan is given by
\[
\pi_{APH} = py + h_x (f_0 - f_i)Ey + h_z [\max(f_i - k, 0) - v_z]Ey + NV_{APH} - C(Ey) \tag{4},
\]
where \(NV_{APH} = p_{APH} \cdot \max(\delta y_{APH} - y, 0) - v_{APH}\). The \(y_{APH}\) is the APH yield of the representative farmer and is calculated by averaging the ten-year county yields from 1996 to 2005. This APH yield is consistent with the former assumption for the representative farm that the mean of the farm yield distribution is equal to the county yield. The coverage level \(\delta\) is arbitrarily set at 70%. If the harvest yield \(y\) is less than the insured yield \(\delta y\), then the farmer gets an indemnity.

\(^5\) Source: [http://www.ces.uga.edu/Agriculture/agecon/printedbudgets.htm](http://www.ces.uga.edu/Agriculture/agecon/printedbudgets.htm)
payment equal to the product of yield shortfall and indemnity price \( p_{APH} \). Indemnity price is the price determined by Risk Management Agency (RMA)\(^6\) of U. S. Department of Agriculture (USDA) multiplied by a price election factor selected by the insured farmer. The representative farmer in this study is assumed to choose a price election factor of 100\%. For cotton insurance in Colquitt in 2006, \( p_{APH} \) is \$0.53/lb. For soybeans in Bulloch in 2006, \( p_{APH} \) is \$5.15/bushel.

The actual APH insurance premium \( v_{APH} \) charged to the insured farmer is set by RMA according to a set of parameters including the farmer’s APH yield and selected coverage level. To make the insurance premium consistent with the data simulated in this study, the actuarially fair premium is used, which is obtained by setting \( v_{APH} = E(p_{APH} \cdot \max(\delta y_{APH} - y, 0)) \).

When CRC is purchased by the representative farmer, the payoff from the hedge portfolio at harvest is

\[
\pi_{CRC} = p y + h_x (f_0 - f_1)E_y + h_z \max( f_1 - k, 0) - v_Z ]E_y + NV_{CRC} - C(E_y)
\]

with \( NV_{CRC} = \max[ \delta \max( f_{CRC0}, f_{CRC1})y_{APH} - f_1 y, 0)] - v_{CRC} \). The coverage level\( \delta \) is still set at 70\%. For cotton, \( f_{CRC0} \) is defined as the daily average settlement price of December contract from January 15 to February 14 of 2006. For soybeans, \( f_{CRC1} \) is the average of daily futures price of the November contract in February, 2006. Premium \( v_{CRC} \) is set as the actuarially fair premium, which is \( v_{CRC} = E(max[ \delta \max( f_{CRC0}, f_{CRC1})y_{APH} - f_1 y, 0)]) \). According to RMA, the subsidized insurance premium can be obtained by multiplying the actuarially fair premium by (1-subsidy factor). Therefore, the effects of premium subsidy on the optimal hedge ratio can be examined by comparing the hedge ratios under an actuarially fair premium with those under the subsidized premium. Results of such comparison will be discussed in the next section.

Hedge Portfolio Including DP, LDP, CCP

Direct payments (DP) are fixed in 2002-2007 and paid annually to eligible farmers enrolled in the government programs. Thus, when DP is added to the hedge portfolio, it increases the harvest value of the hedge portfolio by the amount, \( DP = P_{DP} \cdot y_{DP} \), which is already known at planting time. \( p_{DP} \) and \( y_{DP} \) are the direct payment rate and the base yield fixed from 2002 through 2007. The portfolio with futures, options and DP has payoffs as

\[
\pi = py + h_x (f_0 - f_1)E_y + h_z \max( f_1 - k, 0) - v_Z ]E_y + DP - C(E_y)
\]

Because DP is known at planting, the target payoff in the LPM\(_2\) hedge model with DP in the portfolio is set as \( E(py) + DP \). Since DP increases the actual portfolio income and the target (expected) portfolio income by the same amount, DP has no effect on the LPM\(_2\) hedge ratios.

Loan deficiency payments (LDP) are triggered when the market price is lower than the loan rate set by USDA. Thus, LDP received by the eligible farmer for per-acre yield can be expressed as \( LDP = \max( p_{LDP} - p, 0) \cdot y \). The amount of counter cyclical payments (CCP) paid

\(^6\) RMA provides a premium calculator which can be used to check the price used by RMA at the link [http://www.rma.usda.gov/tools/premcalc.html](http://www.rma.usda.gov/tools/premcalc.html).
to eligible farmers is calculated as \( CCP = \max \{ p_{CCP} - p_{DP} - \max( p_{MYA}, p_{LDP} ), 0 \} \cdot y_{DP} \).

The \( p_{CCP} \) is the target price set in the 2002 farm bill. The \( p_{MYA} \) is the market year average price, which is not known until the end of the crop marketing year. However, in order to evaluate the effects of CCP on the representative farmer’s hedging demand for futures and options at planting, \( p_{MYA} \) is forecasted based on the simulated futures price in this study.

To identify the effects of the LDP and the CCP on hedge ratios, respectively, the two government payments are added to the portfolio (4.4) one by one, with the LDP added first. Such an order is justified by the design of the CCP, which has a target price higher than the loan rate in LDP and would provide extra price-risk protection in conjunction with LDP. The representative farmer’s portfolio, including DP and LDP, is given by

\[
\pi_{LDP} = py + h_x (f_0 - f_1) Ey + h_z [\max( f_1 - k, 0) - v_z ] Ey + DP + LDP - C(Ey) \tag{7}.
\]

When the impacts of CCP are under investigation, the hedge portfolio is given by

\[
\pi_{CCP} = py + h_x (f_0 - f_1) Ey + h_z [\max( f_1 - k, 0) - v_z ] Ey + DP + LDP + CCP - C(Ey) \tag{8}.
\]

By adding net value of APH or CRC (i.e., \( NV_{APH} \) or \( NV_{CRC} \)) into (6), (7) and (8), the hedge portfolios including both insurance and government payments are obtained as follows:

\[
\pi_{DPAPH} = py + h_x (f_0 - f_1) Ey + h_z [\max( f_1 - k, 0) - v_z ] Ey + DP + NV_{APH} - C(Ey) \tag{9}.
\]

\[
\pi_{DPCRC} = py + h_x (f_0 - f_1) Ey + h_z [\max( f_1 - k, 0) - v_z ] Ey + DP + NV_{CRC} - C(Ey) \tag{10}.
\]

\[
\pi_{LDPAPH} = py + h_x (f_0 - f_1) Ey + h_z [\max( f_1 - k, 0) - v_z ] Ey + DP + LDP + NV_{APH} - C(Ey) \tag{11}.
\]

\[
\pi_{LDPCRC} = py + h_x (f_0 - f_1) Ey + h_z [\max( f_1 - k, 0) - v_z ] Ey + DP + LDP + NV_{CRC} - C(Ey) \tag{12}.
\]

\[
\pi_{CCPAPH} = py + h_x (f_0 - f_1) Ey + h_z [\max( f_1 - k, 0) - v_z ] Ey + DP + LDP + CCP + NV_{APH} - C(Ey) \tag{13}.
\]

\[
\pi_{CCPCRC} = py + h_x (f_0 - f_1) Ey + h_z [\max( f_1 - k, 0) - v_z ] Ey + DP + LDP + CCP + NV_{CRC} - C(Ey) \tag{14}.
\]

The specific values of parameters in APH, CRC, DP, LDP and CCP are listed in Zhang (2007).

The LPM2 hedge model used in this study has the objective function as

\[
\text{Min} LPM = \min_{\pi, \underline{\pi}} \int (\pi - \underline{\pi})^2 dF(\pi),
\]

where \( \underline{\pi} \) is the target payoff of the hedge portfolio. The EU hedge model has the objective function given by \( \max_{\pi, \underline{\pi}} \text{EU} = \max_{\pi, \underline{\pi}} \text{EU}(\pi + ow) \), where \( U \) is assumed to be a constant relative risk-averse utility function of the form \( U(\cdot) = \frac{1}{1 - r}(\cdot)^{-r} \). The \( ow \) is the original per-acre farm wealth, which is set to $800 so that it is large enough to guarantee the harvest wealth \( w_1 = ow + \pi > 0 \). The value of the risk aversion coefficient \( r \) is set to 2, the well accepted value by previous studies (Coble et al. 2000, 2004; Mahul, 2003; Wang et al. 1998, 2004; Hauser et al. 2004).

Substituting the values of the hedge portfolios under different scenarios into the two hedge models, numerical optimization methods are used to find the corresponding optimal hedge ratios. By comparing the optimal hedge ratios in the presence of insurance or government payments with the hedge ratios from the baseline scenario, the effects of federal crop insurance or government payments on hedging demand for futures and options can be evaluated under each hedge model. By comparing the hedge ratios under the same scenario across the two hedge models, the different impacts on hedging between the two hedge models can be assessed.
In this presentation, the LPM₂ and EU hedge ratios are compared under scenarios with DP and APH or CRC, or LDP, or LDP and CCP in the portfolios. Inclusion of DP can increase the mean of the payoffs to be positive. The target payoffs in the LPM₂ hedge models for these scenarios with DP are set as the expected value of the hedge portfolio (6), which is \( E(p_y) + DP - C(Ey) \). The impacts of transaction costs associated with hedging on the use of futures have been discussed under the EU hedge model (Coble et al. 2000, 2004; Wang et al. 2004). The effects of transaction costs on optimal LPM₂ hedge ratios are examined here by comparing optimal hedge ratios with and without transaction costs. We also perform sensitivity tests on how LPM₂ hedge ratios vary with the levels of target payoff.

Simulation of Harvest-Time Yield and Price Data

To make the hedge decision at planting, the representative farmer must know the dependence and the harvest-time marginal probability distributions (at least intuitively) of four random variables – farm-level yield, local market price, futures price, and market year average price – in addition to the information known for certain at planting, such as the parameter values determined by USDA or the farm bill for the crop insurance and government payments.

Existing studies generally assume a multivariate normal distribution for the transformed yield and price data, based on the normality test results that each of the transformed historical yield and price data can not be rejected to be marginally normal (Coble et al. 2000, 2004; Wang et al, 2004; Hauser et al. 2004). However, the joint normality assumption is overly restrictive, as random variables with marginal normal distributions can have joint distributions other than multivariate normal. To allow for more flexible dependence structures between yield and price, this study uses the copula function to represent the relationship among yield and price variables. Specifically, two types of copula functions, the Gaussian copula and the Frank copula are used to simulate data. Different types of copulas, together with different parameter values included in the copula function, result in different joint distributions.

Another innovation of this study regarding data simulation is to apply a conditional kernel density approach to simulate farm-level yield with variance contingent on county yield. Due to the short period of the available farm-level yields (a yield record of maximum 10 years for each individual farm) in previous studies, farm-level yields were usually substituted by the rescaled longer county yield series, with the scale to be the ratio of farm yield standard error over county yield standard error. This way of augmenting farm yield assumes that farm-level yield has the same distribution as county-level yield, except for a larger variance. However, historical data show that farm yield variance varies with level of county yield rather than being constant (Zhang 2007).

General Simulation Methods and Historical Data

In Figure 1, a flowchart illustrates the general steps to simulate the random values of four variables, futures price at harvest \( f_j \), harvest local cash price \( p \), market year average price \( p_{MYA} \) and the representative farm’s yield at harvest \( y_j \), which are necessary in order to solve the hedge models. The most important two steps in the simulation are first to generate harvest-time futures price and county yield with the copula method, and then to simulate farm yield.
based on the generated county yield. Detailed simulation algorithms for harvest-time futures price, county yield and farm yield are provided in Zhang (2007). After futures price at harvest is simulated, harvest-time cash price and market year average price will be generated based on their linear relationships with futures price indicated by the historical data.

Farm yield data for cotton in Colquitt county and farm yield data for soybeans in Bulloch county of Georgia from year 1991 to 2000 were obtained from RMA of USDA. Farm-level yield data were used to estimate the empirical conditional farm yield distribution. County-level yield of cotton in Colquitt county from year 1950 to 2005 and county-level yield of soybean in Bulloch county from year 1959 to year 2005 were collected on the website of the NASS of USDA. County yield data were used to estimate the empirical distribution of county yield as well as to estimate the correlation between yield and futures price.

### Figure 1 Simulation Flowchart

Note: $y_c$ is the simulated county yield; $y_f$ is the farm yield; $f_t$ is futures price; $p$ is the cash price; and $p_{MYA}$ is the estimated market year average price.

Daily average futures prices at planting and at harvest were calculated based on the cotton futures data from the New York Board of Trade (NYBOT) from 1978 to 2006 and from soybean futures data from Chicago Board of Trade (CBOT) from 1979 to 2006\(^8\). Harvest-time cash price was approximated by the average price received by Georgia producers collected by USDA from 1978 to 2005 for cotton and from 1979 to 2000 for soybeans\(^9\). Market year average prices from 1978 to 2005 were obtained from NASS of USDA\(^{10}\).

---

\(^7\) Speaking more exactly, this is the correlation between detrended county yield and the difference of logarithm of futures price at harvest and at planting that will be estimated because significant correlation is detected between the two variables for cotton but is not found between detrended county yield and futures price at harvest.

\(^8\) Specifically, cotton price of the December futures contract in March and in November were averaged, respectively; soybean prices of November futures in February and in October were averaged, respectively.


\(^{10}\) Data source: [http://www.nass.usda.gov/QuickStats/Create_Federal_Indv.jsp](http://www.nass.usda.gov/QuickStats/Create_Federal_Indv.jsp)
average prices in 2006 are simulated based on their linear relationships with harvest-time futures prices, respectively. Specifically, parameters in the linear regression of historical cash prices on harvest-time futures prices were estimated by ordinary least square (OLS). Then, cash price in 2006 was simulated by the predicted cash price plus a random shock drawn from the assumed normal residual distribution. Market year average prices were simulated in the same way.

The need to model the distribution of harvest-time yields and prices and the distributional flexibility associated with the copula method motivate this study to use copula simulation. Among the many well-known copulas, two copulas, Gaussian copula and Frank copula were selected to model the dependence between the county yields and the futures prices. The purpose of selecting two copulas is to (i) demonstrate that a variety of joint distributions of yields and prices can exist other than the bivariate normal and (ii) provide a sensitivity analysis of the results. However, this study does not assert that these two copulas make the best fits to the relationships between the sample yields and prices (for copula selection criteria, see Gary, 2002).

Bivariate Gaussian copula takes the form

\[ C_G(u, v) = \Phi_p\left(\Phi^{-1}(u), \Phi^{-1}(v)\right) \]  

where \( \Phi_p \) is the bivariate normal CDF with the Pearson’s coefficient \( \rho \), representing the linear correlation between the two variables \( X_1 \) and \( X_2 \); \( \Phi \) is the normal CDF; \( u \) and \( v \) are variates from two independent Uniform (0, 1) distributions. When the two marginal variables have normal distributions, a bivariate Gaussian copula is equivalent to a bivariate normal CDF, which means \( C_G(\Phi(x_1), \Phi(x_2)) = \Phi_p(x_1, x_2) \) (See proof on p. 113 of Cherubini et al., 2004).

Frank copula is a one-parameter copula function of the form

\[ C_F(u, v) = -\frac{1}{\theta} \ln(1 + \frac{(\exp(-\theta u) - 1)(\exp(-\theta v) - 1)}{\exp(-\theta) - 1}), \theta \neq 0 \]  

When the \( \rho \) in Gaussian copula and \( \theta \) in Frank copula are positive (negative), the marginal distributions coupled by the copulas are positively (negatively) associated.

In this work, the two marginal distributions coupled by the Gaussian copula or Frank copula are the distribution of county yield \( F_y \) and the marginal distribution of the logarithm difference between the futures prices at harvest and at planting, \( F_{dln_f} \). County-level yields must be simulated because the representative farm’s yields are modeled as conditional on the county yields. The marginal distribution of \( (dlnf) \) is used here because, on the one hand, this differentiated variable has a significant negative correlation with county yields. On the other hand, it can be used to generate harvest-time futures price \( f_1 \) when the planting-time futures price \( f_0 \) is known. Significant dependence between \( y_c \) and \( dlnf \) found in historical data determines the sign and magnitude of \( \rho \) in Gaussian copula and \( \theta \) in Frank copula.

Regression analysis on the soybean county-level yield of Bulloch county showed no linear trend in time. Therefore the county yield and farm yield of Bulloch county soybean production were used for the simulation without detrending. Ordinary least squares analysis revealed that cotton yields of Colquitt county from 1950 to 2005 have a significant linear trend. To account for the temporal component, a simple detrending procedure was implemented by
scaling yields from year 1978 through 2005 to year 2006. The detrended county yields were then calculated as:

\[ y_{ct}^{\text{det}} = y_{ct} + \hat{\beta}(2006 - t) \]  

(17)

where \( \hat{\beta} \) is the estimated coefficient for the year variable in the linear trend regression. Although the 6 to 10 years of available farm-level yield is unlikely to represent the underlying yield distribution for each farm (Deng et al., 2007), all the farm-level yield information within a county could be used to estimate the yield distribution of a representative farm in the county by assuming that the yield of the representative farm could be equal to any farm yield harvested in the county in a specific year. By further assuming that every year’s farm yield is multiplicatively conditional on that year’s county yield, random pseudo yields of the representative farm in 2006 were generated as

\[ y_{ft}^{\text{det}} = y_{ct}^{\text{det}} \cdot \frac{y_{ht}}{y_{ct}} \]

where \( t \) is any year of 1991-2000. To simplify notation, detrended county yield and detrended farm yield will be also represented by \( y_c \) and \( y_f \) in the rest of the study.

Yield distributions have been modeled by both parametric methods and nonparametric approaches (Ker and Goodwin, 2000; Wang et al. 2004). This study applied a nonparametric kernel density approach to estimate the empirical distribution of the detrended county yields. In particular, for any \( y_c^* \), the empirical cumulative probability was estimated by

\[
\hat{F}(y_c^*) = P(y_c \leq y_c^*) = \frac{1}{n} \sum_{i=1}^{n} K_h(y - y_{ci})dy
\]

(18)

where \( \frac{1}{n} \sum_{i=1}^{n} K_h(y - y_{ci}) \) is the kernel density estimator. \( y_{ci} \) represents the detrended historical county yields, \( K_h \) is defined as \( K_h(\cdot) = (1/h)K(\cdot/h) \), \( h \) is the bandwidth or smoothing parameter which determines the smoothness of the estimated density, and \( K(\cdot) \) is referred to as the kernel and is usually chosen to be a unimodal probability density function that is symmetric around zero (Wand and Jones, 1995). In this study, the Epanechnikov density function was used as the kernel with \( K(x) = \frac{3}{4}(1 - x^2)1(|x| < 1) \) (19).

Jones et al. (1996) provided a survey for bandwidth selection methods. This study used the bandwidth determined by the Sheather-Jones plug-in method to determine the bandwidth for \( y_c \). \( \hat{F}(y_c^*) \) was calculated by applying Simpson’s rule of numerical integration (Miranda and Fackler, 2002).

Futures prices have been specified by the lognormal distribution in the literature (Coble et al. 2000, 2004; Hauser et al. 2004). Since \( d\ln f = \ln f_1 - \ln f_0 \) calculated with historical cotton and soybean futures data passed several normality tests, this study assumes \( d\ln f \) to be normally distributed with mean and variance determined by the historical data. That is,

\[ F_{d\ln f} = \Phi(\tilde{\mu}_{d\ln f}, \tilde{\sigma}_{d\ln f}) \]

Once planting time futures price \( f_0 \) is known, harvest futures price can be simulated by \( f_1 = f_0 \cdot \exp(d\ln f) \).
Three major steps are involved in simulating interdependent variables with copulas. First, the parameters in the selected copula need to be estimated. This study applies maximum likelihood method to estimate the parameters in the Gaussian copula and the Frank copula. The second step is to generate unit uniform variates based on the estimated copula. Different copulas will have different algorithms to generate unit uniform variates. The third major step is to simulate variates of desired variables (e.g., county yield and transformed futures price) by substituting the generated uniform variates from the second step into the appropriate inverse cumulative distributions.

By following algorithms for the Gaussian and Frank copulas detailed in Zhang (2007), 10,000 pairs of \((y_c, f_i)\) were simulated from the Gaussian copula and from the Frank copula, respectively. In order to investigate the hedging decision under the unbiased futures price assumption, the mean of the simulated futures price was adjusted to equal the futures price at planting time (Wang, et al. 2004). Hedging decisions under biased futures price will also be examined after enlarging or shrinking the mean of the simulated futures by a certain percent.

Both Gaussian copula and Frank copula are applied to simulate yield and price data for cotton. But only the Gaussian copula is used to simulate soybean yield and price data, since no significant dependence between the differenced soybean futures price and county yields is detected. Zero dependence between differenced soybean futures and county yield makes the simulation applying Frank copula no different from the simulation using Gaussian copula because in this case both copulas become the independence copula.

To incorporate this relationship between farm yields and county yields into the simulated yield data, conditional kernel estimation was applied to simulate farm yield based on the generated county yield from Gaussian copula or Frank copula. The simulated farm yields were adjusted to make their mean value equal to the mean of the simulated county yield, because the representative farm is assumed to have mean yield equal to the county yield\(^{11}\).

The derivative-free numerical optimization procedure, the Nelder-Mead algorithm, was used (Miranda and Fackler, 2002) to solve for the minimum LPM\(_2\) hedge ratios. Because the LPM\(_2\) for the hedge portfolio is generally not a globally convex function of the hedge ratios, a grid search approach is used to locate the local minimum LPM\(_2\) hedge ratios first and then based on the grid search results, the global optimal LPM\(_2\) hedge ratio can be quickly found. For the EU hedge model, the BFGS (Broyden, Fletcher, Goldfarb, & Shano) algorithm was used in search for the optimal hedge ratios (Miranda and Fackler, 2002). Model optimization results under various scenarios are reported and discussed in next section.

**COMPARATIVE RESULTS**

**Optimal LPM\(_2\) Hedge Ratios vs Optimal EU Hedge Ratios**

Optimal hedge ratios for the LPM\(_2\) model and the EU model for five scenarios, including direct payments (DP), actual production history (APH), crop revenue coverage (CRC), Loan

\(^{11}\) The \(y_f\) is adjusted by multiplying the ratio of mean \((y_c)/\text{mean}(y_f)\).
deficiency payments (LDP), and counter cyclical payments (CCP) portfolio possibilities, are reported in Tables 1 through 5. The effects of crop insurance policies and government payments on the optimal hedging in futures and put options contracts from the two models appear consistent in terms of the signs of the effects. Thus, whether the LPM\textsubscript{2} hedge model or the EU hedge model is used to evaluate such effects, the directions of the impacts are observed to be the same.

The magnitudes of the optimal hedge ratios in futures contracts from the LPM\textsubscript{2} model are all smaller than the futures hedge ratios suggested in the EU model. In other words, the producer hedges a smaller percentage of his expected production if he uses an LPM\textsubscript{2} model to estimate the optimal hedge ratio instead of EU model. These results do not reject the hypothesis that model misspecification may be another reason for the fact that the observed hedge in futures by farmers is less than the predicted hedge by the EU model. If the hedging objective is to minimize perceived risk to the farming income, but not to maximize the expected utility, then the crop farmers would appear to under-hedge when they are judged by the EU model.

Results presented in Tables 1 to 5 suggest that the hedge ratios in put options under the LPM\textsubscript{2} hedge model exceed or equal the optimal hedge ratios estimated from the EU hedge model. That put options are preferred by the LPM\textsubscript{2} model could be explained by the evidence that the asymmetric payoff distribution of put options is consistent with the skewed distributions of the hedge portfolios that the LPM\textsubscript{2} hedge model generally yields.

By comparing the basic distributional statistics of the hedge portfolios, it can be seen that when the means of the hedge portfolio are the same, the hedge portfolios from the LPM\textsubscript{2} model always have a larger right skewness, a higher minimum payoff, and a higher maximum payoff than the portfolios from the EU hedge model. Although the EU hedge model yields portfolios with a smaller standard deviation than the LPM\textsubscript{2} model, any producer who desires to hedge against low income events may prefer the hedging strategies resulting from the LPM\textsubscript{2} hedge.

**Effects of Insurance Coverage, Premium Subsidy**

Sensitivity analysis in Table 6 shows that the effects of crop insurance policy (APH or CRC) on the futures and put options hedge ratios are amplified as the coverage level increases. That is, the higher the coverage level, the stronger the effects the crop insurance has on the hedge ratios. For example, as the coverage level increases from 60\% to 70\% and from 70\% to 80\%, the optimal portfolio consistently sells more and more futures contract and buy less and less options (sell options can be viewed as buy negative options).

Table 6 also shows that effects of subsidy to hedge ratios are relatively small. With the maximum subsidy rate, the hedge ratios are only affected by as little as 0.05. This suggests that crop insurance premium subsidies may not have notably impacted the hedging decision of the crop farmers, although such subsidies have already amounted to billions of dollars per year. Conversely, government subsidies in the form of LDP and CCP appear to greatly change the hedging demands by these cotton and soybean producers.

**Transaction Costs and Target Payoff**

The optimal LPM\textsubscript{2} hedge ratios with transaction costs on futures contracts are reported in Table 7. The effects of transaction cost can be obtained by comparing Table 7 with Tables 1 to 5.
for the corresponding scenarios and crops. Consistent with findings in Wang et al. (2004), the introduction of transaction cost decreases the hedging demand for futures.

Table 8 shows that increasing the target payoff in the LPM2 hedge model increases the optimal hedge ratio in futures slightly. But the effects on put options seem to depend on the specific crop/country combination and could lead to a slightly higher or a lower use of put options. The insensitivity of the optimal LPM2 hedge ratios to the target income indicates that even if the target income can not be determined precisely, the optimal hedge ratios would not be substantively different.

CONCLUSIONS

The insurance policies and government payments evaluated in the EU model have the same directional influences as they are in the LPM2 hedge model. However, the LPM2 model may be preferred by crop farmers, because the payoff distribution of the hedge portfolio from the LPM2 hedge model has a higher minimum value, a higher maximum value and a higher right skewness compared to the hedge portfolio of equal mean from the EU hedge model. Thus, if it is lower income that producers desire to hedge against, the LPM2 hedge model is more appropriate than the EU hedge model to evaluate the portfolio effects of government payments and crop insurance on the expected hedging behavior of crop farmers.

Compared to the EU hedge model, the LPM2 model also yields a consistently smaller optimal proportion of the expected production to be hedged with futures and put options. Such a result supports the hypothesis that model misspecification is a likely reason for the fact that the observed hedging in futures by farmers is less than the predicted optimal hedge by using the EU model. If the hedging objective is to minimize the perceived risk to (lower) farm income, but not to maximize the expected utility, then crop farmers would appear to under-hedge from the perspective of the EU model.

The finding that the LPM2 hedge model generally yields a lower hedge ratio in futures but a higher hedge ratio in put options compared to the EU hedge model may result from the reality that the asymmetric payoff distribution of put options is consistent with the more skewed distributions of the hedge portfolios that the LPM2 hedge model generally yields. Transaction costs associated with trading in futures are assumed to be composed of commissions and interest forgone on margin deposits. By model design, the commissions would not affect the optimal hedge ratios. However, transaction costs incurred by the interest loss of margins decreases the hedging demand for futures.

Sensitivity tests on effects of insurance coverage levels on hedge ratios suggest that the higher the coverage level, the stronger the effects that the crop insurance policies will have on the hedge ratios. The relatively small effects of insurance premium subsidy rates on the hedge ratios indicate that, compared with government subsidies in forms of LDP and CCP, the crop insurance premium subsidies may have little impact on the hedging decisions of the crop farmers even though such subsidies amount to billions of dollars every year lately. The robustness of the optimal LPM2 hedge ratios to the target incomes indicates that even if the target income can not be determined precisely, the optimal hedge ratios would vary minimally.
This study makes two primary contributions to agricultural risk management literature. First, we developed a downside risk hedge model, the LPM$_2$ hedge model, which is more suitable model than the existing EU hedge model, to evaluate the interactions of government and private risk management tools used by U.S. crop farmers. Second, this study initiated the application of the conditional kernel density method and the copula approach in simulating the crop prices and yields. Such simulation techniques can be extended to model joint distributions of various variables of research interest.

SELECTED REFERENCES


Table 1. Optimal Hedge Ratios from LPM$_2$ and EU Model (Baseline, Futures and Options plus direct payments, FOD)

<table>
<thead>
<tr>
<th>Futures+Options+DP</th>
<th>Cotton Gaussian Copula</th>
<th></th>
<th>Cotton Frank Copula</th>
<th></th>
<th>Soybean Gaussian Copula</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LPM$_2$</td>
<td>EU</td>
<td>LPM$_2$</td>
<td>EU</td>
<td>LPM$_2$</td>
</tr>
<tr>
<td>$h_x$</td>
<td>0.14</td>
<td>0.18</td>
<td>0.32</td>
<td>0.33</td>
<td>0.35</td>
</tr>
<tr>
<td>$h_z$</td>
<td>0.26</td>
<td>0.23</td>
<td>0.16</td>
<td>0.16</td>
<td>0.27</td>
</tr>
<tr>
<td>Mean</td>
<td>44.06</td>
<td>44.06</td>
<td>46.24</td>
<td>46.24</td>
<td>9.27</td>
</tr>
<tr>
<td>Std Dev</td>
<td>163.40</td>
<td>163.12</td>
<td>163.84</td>
<td>163.73</td>
<td>76.54</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.147</td>
<td>0.132</td>
<td>0.118</td>
<td>0.112</td>
<td>0.712</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.459</td>
<td>0.439</td>
<td>0.362</td>
<td>0.361</td>
<td>0.753</td>
</tr>
<tr>
<td>Min</td>
<td>-445.03</td>
<td>-449.66</td>
<td>-497.47</td>
<td>-500.19</td>
<td>-162.07</td>
</tr>
<tr>
<td>Q1</td>
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<td>-59.03</td>
<td>-59.28</td>
<td>-59.07</td>
<td>-49.21</td>
</tr>
<tr>
<td>Q3</td>
<td>148.97</td>
<td>148.55</td>
<td>150.20</td>
<td>149.87</td>
<td>51.89</td>
</tr>
<tr>
<td>Max</td>
<td>882.65</td>
<td>847.24</td>
<td>834.67</td>
<td>832.09</td>
<td>473.56</td>
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Note: $h_x$, $h_z$ are hedge ratios in futures and put options. A positive $h_x$ means sell futures at planting time and a positive $h_z$ means buy put options at planting time.
Table 2. Optimal Hedge Ratios from LPM and EU Model (adding protection of Actual Production History, APH, coverage)

<table>
<thead>
<tr>
<th>Futures+Options+DP +APH</th>
<th>Cotton Gaussian Copula</th>
<th>Cotton Frank Copula</th>
<th>Soybean Gaussian Copula</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LPM₂</td>
<td>EU</td>
<td>LPM₂</td>
</tr>
<tr>
<td>$h_x$</td>
<td>0.27</td>
<td>0.34</td>
<td>0.40</td>
</tr>
<tr>
<td>$h_z$</td>
<td>0.17</td>
<td>0.16</td>
<td>0.13</td>
</tr>
<tr>
<td>Mean</td>
<td>44.06</td>
<td>44.06</td>
<td>46.24</td>
</tr>
<tr>
<td>Std Dev</td>
<td>143.94</td>
<td>143.57</td>
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</tr>
<tr>
<td>Skewness</td>
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<td>0.637</td>
</tr>
<tr>
<td>Kurtosis</td>
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<td>0.230</td>
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<tr>
<td>Min</td>
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<tr>
<td>Q1</td>
<td>-69.73</td>
<td>-69.38</td>
<td>-69.50</td>
</tr>
<tr>
<td>Q3</td>
<td>137.27</td>
<td>137.75</td>
<td>138.62</td>
</tr>
<tr>
<td>Max</td>
<td>847.50</td>
<td>833.87</td>
<td>807.14</td>
</tr>
</tbody>
</table>

Note: $h_x$, $h_z$ are hedge ratios in futures and put options. Positive $h_x$ means sell futures at planting time and positive $h_z$ means buy put options at planting time.
Table 3. Optimal Hedge Ratios from LPM and EU Model (adding protection of crop revenue coverage, CRC)

<table>
<thead>
<tr>
<th>Futures+Options+DP+CRC</th>
<th>Cotton Gaussian Copula</th>
<th>Cotton Frank Copula</th>
<th>Soybean Gaussian Copula</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LPM$_2$</td>
<td>EU</td>
<td>LPM$_2$</td>
</tr>
<tr>
<td>$h_x$</td>
<td>0.38</td>
<td>0.44</td>
<td>0.49</td>
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<tr>
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</tr>
<tr>
<td>Mean</td>
<td>44.06</td>
<td>44.06</td>
<td>46.24</td>
</tr>
<tr>
<td>Std Dev</td>
<td>137.39</td>
<td>136.98</td>
<td>138.43</td>
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<tr>
<td>Skewness</td>
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<td>0.794</td>
</tr>
<tr>
<td>Kurtosis</td>
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<td>0.504</td>
<td>0.447</td>
</tr>
<tr>
<td>Min</td>
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<td>-186.97</td>
</tr>
<tr>
<td>Q1</td>
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<td>Q3</td>
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</tr>
<tr>
<td>Max</td>
<td>828.41</td>
<td>816.23</td>
<td>792.02</td>
</tr>
</tbody>
</table>

Note: $h_x$, $h_z$ are hedge ratios in futures and put options. Positive $h_x$ means sell futures at planting time and positive $h_z$ means buy put options at planting time.
Table 4. Optimal Hedge Ratios from LPM and EU Model (adding LDP)

<table>
<thead>
<tr>
<th>DP+Futures+Options + LDP</th>
<th>Cotton Gaussian Copula</th>
<th>Cotton Frank Copula</th>
<th>Soybean Gaussian Copula</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LPM&lt;sub&gt;2&lt;/sub&gt;</td>
<td>EU</td>
<td>LPM&lt;sub&gt;2&lt;/sub&gt;</td>
</tr>
<tr>
<td>h&lt;sub&gt;x&lt;/sub&gt;</td>
<td>0.15</td>
<td>0.22</td>
<td>0.33</td>
</tr>
<tr>
<td>h&lt;sub&gt;z&lt;/sub&gt;</td>
<td>-0.23</td>
<td>-0.28</td>
<td>-0.33</td>
</tr>
<tr>
<td>Mean</td>
<td>60.91</td>
<td>60.91</td>
<td>63.17</td>
</tr>
<tr>
<td>Std Dev</td>
<td>165.69</td>
<td>165.36</td>
<td>166.21</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.078</td>
<td>0.060</td>
<td>0.056</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.387</td>
<td>0.357</td>
<td>0.286</td>
</tr>
<tr>
<td>Min</td>
<td>-429.06</td>
<td>-435.36</td>
<td>-482.65</td>
</tr>
<tr>
<td>Q1</td>
<td>-43.94</td>
<td>-43.70</td>
<td>-44.91</td>
</tr>
<tr>
<td>Q3</td>
<td>169.98</td>
<td>170.76</td>
<td>170.95</td>
</tr>
<tr>
<td>Max</td>
<td>897.51</td>
<td>885.84</td>
<td>849.65</td>
</tr>
</tbody>
</table>
Table 5. Optimal Hedge Ratios from LPM and EU Model (adding LDP and CCP)

<table>
<thead>
<tr>
<th>DP+Futures+Options</th>
<th>Cotton Gaussian Copula</th>
<th>Cotton Frank Copula</th>
<th>Soybean Gaussian Copula</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LPM₂</td>
<td>EU</td>
<td>LPM₂</td>
</tr>
<tr>
<td>hₓ</td>
<td>-.34</td>
<td>-.17</td>
<td>-.19</td>
</tr>
<tr>
<td>hᵧ</td>
<td>-.02</td>
<td>-.16</td>
<td>-.09</td>
</tr>
<tr>
<td>Mean</td>
<td>125.91</td>
<td>125.91</td>
<td>128.06</td>
</tr>
<tr>
<td>Std Dev</td>
<td>167.36</td>
<td>166.38</td>
<td>168.38</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.112</td>
<td>0.061</td>
<td>0.087</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.537</td>
<td>0.437</td>
<td>0.355</td>
</tr>
<tr>
<td>Min</td>
<td>-387.01</td>
<td>-394.46</td>
<td>-409.92</td>
</tr>
<tr>
<td>Q1</td>
<td>20.23</td>
<td>20.64</td>
<td>17.96</td>
</tr>
<tr>
<td>Q3</td>
<td>235.57</td>
<td>234.64</td>
<td>236.38</td>
</tr>
<tr>
<td>Max</td>
<td>1033.24</td>
<td>977.00</td>
<td>955.29</td>
</tr>
<tr>
<td>Insurance guarantee level (Premium subsidy rate)</td>
<td>Cotton Gaussian Copula</td>
<td>Cotton Frank Copula</td>
<td>Soybean Gaussian Copula</td>
</tr>
<tr>
<td>------------------------------------------------</td>
<td>------------------------</td>
<td>---------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>DP + Futures + Options + APH</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60%</td>
<td>0.225</td>
<td>0.197</td>
<td>0.416</td>
</tr>
<tr>
<td>70%</td>
<td>0.268</td>
<td>0.174</td>
<td>0.436</td>
</tr>
<tr>
<td>80%</td>
<td>0.319</td>
<td>0.153</td>
<td>0.461</td>
</tr>
<tr>
<td>DP + Futures + Options + APH with premium subsidy</td>
<td>0.222</td>
<td>0.197</td>
<td>0.403</td>
</tr>
<tr>
<td>60% (0.64)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>70% (0.59)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80% (0.48)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DP + Futures + Options + CRC</td>
<td>0.301</td>
<td>0.036</td>
<td>0.564</td>
</tr>
<tr>
<td>60%</td>
<td></td>
<td></td>
<td>- .143</td>
</tr>
<tr>
<td>70%</td>
<td>0.381</td>
<td>- .097</td>
<td>0.643</td>
</tr>
<tr>
<td>80%</td>
<td>0.477</td>
<td>- .280</td>
<td>0.734</td>
</tr>
<tr>
<td>DP + Futures + Options + CRC with premium subsidy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>60% (0.64)</td>
<td>0.298</td>
<td>0.030</td>
<td>0.551</td>
</tr>
<tr>
<td>70% (0.59)</td>
<td>0.381</td>
<td>- .112</td>
<td>0.630</td>
</tr>
<tr>
<td>80% (0.48)</td>
<td>0.482</td>
<td>- .307</td>
<td>0.723</td>
</tr>
</tbody>
</table>

Note: premium subsidy rates are obtained from the actuarial files of risk management agency (RMA), USDA.
Table 7 Effects of Transaction Cost of Trading Futures and Options on the Optimal LPM2 Hedge Ratios

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Cotton Gaussian Copula</th>
<th>Cotton Frank Copula</th>
<th>Soybean Gaussian Copula</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h_x$</td>
<td>$h_z$</td>
<td>$h_x$</td>
</tr>
<tr>
<td>Futures + Options +DP (FOD)</td>
<td>0.11</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>FOD + APH</td>
<td>0.25</td>
<td>0.21</td>
<td>0.38</td>
</tr>
<tr>
<td>FOD + CRC</td>
<td>0.36</td>
<td>-0.07</td>
<td>0.47</td>
</tr>
<tr>
<td>FOD + LDP</td>
<td>0.13</td>
<td>-0.19</td>
<td>0.31</td>
</tr>
<tr>
<td>FOD + APH + LDP</td>
<td>0.27</td>
<td>-0.29</td>
<td>0.40</td>
</tr>
<tr>
<td>FOD + CRC + LDP</td>
<td>0.40</td>
<td>-0.61</td>
<td>0.50</td>
</tr>
<tr>
<td>FOD + LDP + CCP</td>
<td>-0.32</td>
<td>-0.06</td>
<td>-0.17</td>
</tr>
<tr>
<td>FOD + APH + LDP + CCP</td>
<td>-0.13</td>
<td>-0.17</td>
<td>-0.03</td>
</tr>
<tr>
<td>FOD + CRC + LDP + CCP</td>
<td>0.02</td>
<td>-0.58</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Note: Transaction cost is calculated as the interest forgone on the margin. Margin requirement is $900/contract for cotton futures contract and is $750/contract for soybean futures. The LPM$_2$ hedge ratios are not affected by DP since DP increases the actual income and the expected income by the same amount.
Table 8 Optimal LPM2 Hedge ratios at Varying Target Income Levels in Base Scenario

<table>
<thead>
<tr>
<th>Target Income</th>
<th>Cotton Gaussian Copula</th>
<th>Cotton Frank Copula</th>
<th>Soybean Gaussian Copula</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h_x$</td>
<td>$h_z$</td>
<td>$h_x$</td>
</tr>
<tr>
<td>$0.5^*(E_{py_t} - C)$</td>
<td>0.12</td>
<td>0.262</td>
<td>0.29</td>
</tr>
<tr>
<td>$0.8^*(E_{py_t} - C)$</td>
<td>0.13</td>
<td>0.261</td>
<td>0.31</td>
</tr>
<tr>
<td>$E_{py_t} - C$</td>
<td>0.14</td>
<td>0.259</td>
<td>0.32</td>
</tr>
<tr>
<td>$1.2^*(E_{py_t} - C)$</td>
<td>0.15</td>
<td>0.257</td>
<td>0.33</td>
</tr>
<tr>
<td>$1.5^*(E_{py_t} - C)$</td>
<td>0.16</td>
<td>0.255</td>
<td>0.34</td>
</tr>
</tbody>
</table>