THE PERFORMANCE OF CONDITIONAL VALUE-AT-RISK VERSUS VALUE-AT-RISK IN RANKING CROP INSURANCE ALTERNATIVES

Abstract

This study examines the use of conditional value-at-risk (CVaR) as a measure for evaluating risky alternatives. CVaR has been shown to have a number of advantages compared to the more traditionally applied value-at-risk in decision settings that involve choices among truncated revenue distributions. A conceptual presentation illustrates important issues in the use of CVaR and its relationship to VaR. An empirical model evaluates their performance for crop insurance decisions faced by grain farmers. The results show that use of VaR eliminates risk-efficient choices that CVaR retains.

Keywords: conditional value-at-risk, crop insurance, risk-return tradeoffs, stochastic dominance, value-at-risk

The pervasiveness of decision-making under risk has led to continual efforts to improve methods for comparing risky alternatives. Seminal developments include construction of risk-return efficient sets, use of stochastic dominance techniques, and improvements in representations of utility functions. Recently, value-at-risk (VaR) techniques have become a standard for risk analysis, being used extensively by financial institutions (Saunders) and in non-financial settings (Bodnar, et.al.). Agricultural economists also have begun to use VaRs in futures market research (see Manfredo and Leuthold for a review) and to evaluate crop insurance alternatives (e.g., Gloy and Baker (2001); Schnitkey, Miranda, and Irwin). The popularity of value-at-risk techniques arises in part from the straightforward interpretation of a VaR value as the point on a cumulative probability distribution corresponding to a specified probability. The use of VaRs as “risk-limits” has direct implication in financial settings where tolerance or limit for loss exposure can be expressed in probabilistic terms, or where insurance is being considered for mitigating risks that occur with some specified probability.

While popular, VaR has a number of properties which are problematic in typical applications. For example, rankings made using VaR in a risk-return framework are not
necessarily consistent with a rational utility function (Kaplanski and Kroll). When used to rank choices using utility models, VaR use can lead to the elimination of otherwise desirable insurance that provides protection of low-probability, high-loss events (Basak and Shapiro). VaR also is unstable and difficult to work with when distributions are not well-behaved (Rockafellar and Uryasev). Moreover, VaR does not provide a measure of losses beyond the threshold amount for the probability. These issues have led to the development of conditional value-at-risk (CVaR) as a measure to evaluate risky alternatives. CVaR is the conditional expected value below a point that occurs with a given probability. Thus, VaR primarily provides information related to a given frequency event – e.g., that revenues are below $500 only 1% of the time – while CVaR provides information related to the severity in cases that exceed the VaR – e.g., that the expected value in the worst 1% of cases is $300.

This study compares VaR and CVaR as risk measures when evaluating crop insurance alternatives. Crop insurance represents the principal means the Federal government uses in providing crop loss assistance to farmers and was meant to replace ad-hoc disaster assistance programs (Glauber and Collins). Since 2000, total premium on Federal crop insurance programs have exceeded $2.5 billion per year and total liability per year has been above $30 billion (USDA). These factors, along with the expansion of the crop insurance program since 1990, have caused crop insurance to be an important class of problems evaluated by agricultural economists. Moreover, crop insurance policies often truncate the crop revenue distribution and VaR risk measures are known to have difficulties in properly evaluating alternatives with non-smooth density distributions (Agarwal and Naik, Basak and Shapiro, Longin). Hence, crop insurance evaluation represents a case in which use of VaR may be problematic.
The analysis proceeds as follows. First, the VaR risk measure is introduced and used to evaluate alternatives with and without crop insurance. This evaluation shows that use of VaR can result in the selection of alternatives that are risk-return dominated by other alternatives. The CVaR risk measure then is introduced and shown to mitigate the risk-return dominance problem. Next, linkages are made between efficient sets developed using VaR and CVaR risk measures and efficient sets developed using stochastic dominance techniques. This analysis shows that an efficient set developed using a CVaR risk measure is a subset of the second order stochastic dominant (SOSD) efficient sets, while an efficient set developed using VaR is not necessarily a subset of the SOSD set. As a result, efficient sets constructed by VaR can include alternatives that are dominated by other alternatives, while the CVaR efficient set will not include dominated alternatives. This result again demonstrates that CVaR use is preferred to VaR use.

Then, CVaR and VaR risk measures are compared in an empirical evaluation of crop insurance alternatives on a central-Illinois farm raising corn and soybeans in a 50-50 crop rotation. For this application, the alternatives are divided into efficient sets using SOSD and also by a risk-return framework using VaR and CVaR as risk measures. Compared to SOSD and CVaR efficient sets, VaR efficient sets eliminate alternatives that have the most reduction in downside risk. From this empirical evaluation, as well as from the conceptual background, it is concluded that CVaR is a superior risk measure compared to VaR for evaluating crop insurance alternatives.

**Values-at-Risk (VaRs)**

For a specific risky alternative, VaR of some value defines a given probability level on the cumulative revenue distribution. The VaR at a probability level is often expressed as:

\[ 	ext{VaR}_p = \text{VaR} \text{ at probability level } p \]
\[ a = F(VaR_a) \]  \hspace{1cm} (1)

where \( F() \) is the cumulative distribution function.² Equivalently,

\[ VaR_a = F^{-1}(a). \]  \hspace{1cm} (2)

where \( F^{-1}() \) represents the inverse cumulative distribution function. \( VaR \) has a straight-forward interpretation. For example, a \( VaR_{05} \) of $235 means that five percent of the time revenue will be less than $235.

When given \( VaRs \) for several risky alternatives, decision-makers are likely to use \( VaRs \) in conjunction with profitability measures to choose among risky alternatives. Kaplanski and Kroll examine the use of \( VaRs \) in a decision setting in which alternatives are ranked using an risk-return framework. Under this approach, each alternative is ranked according to its expected value (\( E \)) and \( VaR_a \) combined into a single scale using some risk parameter (\( \beta \)):

\[ E + \beta \cdot VaR_a \]  \hspace{1cm} (3)

Kaplanski and Kroll show that rankings using (3) will not necessarily result in risk efficient alternatives being ranked higher than inefficient alternatives.

Problems with \( VaR \) are illustrated by examining the impacts that crop revenue insurance typically has on per acre revenue.³ Figure 1 portrays two alternatives: one without revenue insurance and one with insurance. The alternative without revenue insurance has a distribution \( F_{NI}(r) \) that defines the cumulative probability for different revenue (\( r \)). If revenue is below \( r^* \), the insurance policy pays an indemnity (\( ip \)) that causes revenue plus the indemnity payment to equal \( r^* \) (i.e., \( ip = r^* - r \)). The insurance policy is assumed to be actuarially fair. As a result, an actuarially fair insurance premium (\( q \)) must be paid (i.e., \( q = \int_0^{r^*} (r^* - r) f(r) dr \) where \( f(r) \) is the probability density function of the no insurance case). The cumulative distribution of the alternative with revenue insurance is:
\[ F_f(r) = \begin{cases} 
0 & \text{for } r < r^* - q \\
F_{NI}(r - q) & \text{for } r = r^* - q 
\end{cases} \quad (4) \]

In essence, the revenue insurance places a floor on revenue at \( r^* - q \) (see Figure 1). All individuals with continuous utility functions who are risk averse prefer the insurance alternative because it has the same expected value as the no insurance alternative while eliminating all revenue below \( r^* - q \). Thus, one would expect a reasonable risk measure to indicate that the insurance alternative has lower risk as well.

However, the VaR risk measure does not indicate that the insurance alternative has lower risk for all \( a \). For \( a < a^* = F_{NI}(r^* - q) \), \( VaR_a \) for the insurance alternative is greater than the \( VaR_a \) for the no insurance case. For \( a > a^* \), however, \( VaR_a \) for the insurance alternative is less than \( VaR_a \) for the no insurance alternative, indicating that the no insurance alternative is the less risky alternative (see Figure 1). The no insurance alternative will be selected by a decision-maker who uses the expected value-risk framework outlined in equation (3) with \( VaR_a \) greater than \( a^* \).

Another potential use of \( VaR \) is in a model that maximizes either expected utility or expected profits while imposing a constraint on the lowest acceptable level of \( VaR_a \). Basak and Shapiro examine the impacts of these constraints in models that maximize the expected utility of terminal wealth. They find that decision-makers who meet a \( VaR \) constraint can choose alternatives that do not insure against high-loss events that have low probability of occurring. This problem also can be examined in Figure 1. If a model's \( VaR \)-constraint is above \( a^* \), the insurance alternative could be eliminated from consideration even though all risk averse individuals prefer the insurance alternative.\(^4\)
Conditional Values-at-Risks (CVaRs)

Problems with VaRs have lead to the development of another risk measure variously termed the accumulated VaR (Kaplanski and Kroll), beyond VaR (Longin), conditional VaR (Uryasev), and tail VaR (Aztner, et.al. (1999)). Herein the measure is referred to as the conditional VaR (CVaR). As defined by Kaplanski and Kroll, CVaR is the integral of VaR up to $a$:

$$CVaR_a = \left( \int_0^a VaR_y \, dy \right) / a = \left( \int_0^a F^{-1}(y) \, dy \right) / a$$  \hspace{1cm} (5)

Equation (5) provides the intuition for calling this measure the accumulated VaR and is useful for showing linkages between CVaR and stochastic dominance. An equivalent definition of CVaR is

$$CVaR_a = \int_{-\infty}^{VaR_a} r \cdot f(r) \, dr / a$$  \hspace{1cm} (6)

which is Longin’s definition of CVaR. Equation (6) expresses CVaR directly as the conditional expected value over the range from -8 to VaR$_a$. These revenues account for $a$ probability of the worst outcomes.

CVaR overcomes many of the problems associated with VaR. Kapalanski and Kroll show that rankings using CVaRs are consistent with a rational expected utility function. Basak and Shapiro show that use of CVaRs as a constraint is much more likely to result in the insurance of low-probability, high-loss events than is a VaR constraint.

Alternatives in Figure 1 illustrate the superiority of CVaR over VaR. For $a < a^*$, CVaR for the insurance alternative equals $r^* - q$ and, because all revenues for the no-insurance case are below $r^* - q$, CVaR for the no insurance alternative will be below CVaR for the insurance case. As a result, both CVaRs and VaRs indicate that the insurance alternative is the least risky alternative for $a < a^*$. For $a > a^*$ but less than 100%, CVaR$_a$ for the insurance case will be
greater than $CVaR_a$ for the no insurance case, indicating that the insurance alternative is the least risky alternative. In this a range, $VaR$ indicates the no insurance case is the least risky case. When $a$ equal to 100%, $CVaR_{100}$ equals the expected value of the alternatives and, since both alternatives have the same expected value, $CVaR$ is the same for both alternatives. Therefore, $CVaR$ indicates that the insurance alternative is the least risky alternative over a much larger range of $a$ than for $VaR$.

**Stochastic Dominance, VaR, and CVaR**

Efficient sets can be developed using $VaR$ and $CVaR$ as measures of risk in a general expected value-risk framework. Alternative $x$ dominates alternative $y$ if both of the following conditions are met with at least one holding with strict inequality:

\[
E(x) = E(y) \quad (7-a)
\]

\[
RISK(x) = RISK(y) \quad (7-b)
\]

where $E(i)$ is the expected value of alternative $i$ and $RISK(i)$ is either the $VaR$ or $CVaR$ measure for alternative $i$. $VaR$ dominance of $x$ over $y$ is denoted as $x \ D_{VaR_a} \ y$ where $VaR_a$ is used as the risk measure in (7-b). Similarly $x \ D_{CVaR_a} \ y$ means that $x$ dominates $y$ using $CVaR_a$ in (7-b).

$VaR$ efficient sets developed using (7) exhibit a relationship with first order stochastic dominance (FOSD) efficient sets (Kapalanski and Kroll). Alternative $x$ will exhibit FOSD over another alternative $y$, denoted as $x \ D_{FOSD} \ y$, when the cumulative distribution of the first alternative ($F_x(r)$) lies to the right of the cumulative distribution for the second distribution ($F_y(r)$):

\[
F_x(r) = F_y(r) \quad (8)
\]
for all $r$ in the domain $[a, b]$ (Ingersoll). The requirement in (8) can be alternatively stated relative to the inverse cumulative function:

$$F_x^{-1}(t) \geq F_y^{-1}(t)$$

(9)

for all $t$ between 0 and 1. Note that $F_x^{-1}(t)$ is the definition of $VaR_a$ at $a$ equal to $t$ and that (9) implies that the second equation of $VaR$ dominance (i.e., equation 7-b) holds. Also, $x \overset{D}{\geq}_{FOSD} y$ implies that the expected value of $x$ is greater than the expected value of $y$ (Levy), meaning that the first equation of $VaR$ dominance (7-a) also holds. Therefore, FOSD implies $VaR$ dominance:

$$x \overset{D}{\geq}_{FOSD} y \Rightarrow x \overset{D}{\geq}_{VaR_a} y$$

(10)

The relationship in (10) implies that $VaR$ efficient sets are subsets of the FOSD efficient set.

Moreover, second order stochastic dominance (SOSD) implies $CVaR$ dominance (Kapalanski and Kroll). Alternative $x$ exhibits SOSD dominance over alternative $y$, denoted as $x \overset{D}{\geq}_{SOSD} y$, if and only if:

$$\int_a^t F_x(r)dr \leq \int_a^t F_y(r)dr$$

(11)

for all $t$ (Ingersoll). Equation 11 implies that

$$\int_0^a F_x^{-1}(t)dt \geq \int_0^a F_y^{-1}(t)dt$$

(12)

for all $a$ between 0 and 1. The expression on the left and right hand sides of equation (12), respectively, represent $CVaR_a$ for alternative $x$ and $y$. Therefore, the second equation of $CVaR$ dominance (i.e., equation 7-b) is met. Moreover, $x \overset{D}{\geq}_{SOSD} y$ implies that the expected value of $x$ is greater than the expected value of $y$ (Levy), meaning that the first equation of $CVaR$ dominance (7-a) also holds. Therefore, SOSD implies $CVaR$ dominance:

$$x \overset{D}{\geq}_{SOSD} y \Rightarrow x \overset{D}{\geq}_{CVaR_a} y$$

(13)
In an empirical setting, the relationship in (13) implies that the CVaR efficient set is a subset of the SOSD efficient set. Hence, an alternative that is identified as CVaR efficient also will be SOSD efficient. However, an alternative that is identified as SOSD efficient does not have to be CVaR efficient.

CVaR efficient sets have some desirable properties for individuals who maximize the expected value under continuous utility functions and who are risk averse. The efficient CVaR efficient set will only include SOSD efficient alternatives. This differs from use of VaR because VaR efficient sets will not necessarily include only SOSD efficient alternatives. If there is only one SOSD efficient alternative among all alternatives, the CVaR efficient set will consist only of that one alternative. When the SOSD set is larger, then there is a possibility that use of CVaR efficiency will eliminate some alternatives that a risk averse individual may find optimal. The extent to which this result occurs is an empirical issue that is examined in the following sections.

**Empirical Application and Revenue Distributions**

The empirical evaluation that follows examines the extent to which CVaR and VaR efficient sets span the risk and return characteristics that risk averse individuals would consider when choosing among risky alternatives. Efficient sets are derived using SOSD techniques and by using CVaR and VaR as risk measures in equations (7). CVaR and VaR efficient sets then are compared to the SOSD efficient set. The main purpose of these comparisons is to determine if alternatives in the CVaR and VaR efficient sets provide a reasonable approximation of the risk-return characteristics of alternatives any risk averse individual would consider as indicated by alternatives in the SOSD efficient set. The measure that more closely approximates the full breadth of risk-return characteristics in the SOSD set is judged as having the highest efficacy.
CVaR and VaR efficient sets are generated at different a values to examine the influence of a-level on results.

Comparisons are done by generating revenue distributions for a central-Illinois farm that raises 50% corn and 50% soybeans, a typical rotation of Midwest grain farmers. Gross revenues are simulated for alternatives that use different insurance products at different coverage levels for both corn and soybeans. Revenue is stated on a per acre basis and includes crop revenue from each crop. Per acre crop revenue from corn equals the cash price at harvest, \( p_c \), times the yield per acre, \( y_c \) (revenue from soybeans equals \( p_s \cdot y_s \)). Besides crop revenue, revenue from loan deficiency payments, \( ldp_c \) and \( ldp_s \), and counter-cyclical payments, \( ccp_c \) and \( ccp_s \) are included because proceeds from these programs may have risk mitigating impacts that substitute for crop insurance proceeds. Per acre revenue equals:

\[
 r(i,j,k,l) = \left[ \sum_{o \in \{c,s\}} \left( p_o \cdot y_o + ldp_o + ccp_o + ip_{o,i,j} - q_{o,i,j} \right) \right] / 2
\]

(14)

where \( r(i,j,k,l) \) is revenue given that crop insurance product \( i \) at coverage level \( j \) is used to insure corn and crop insurance product \( k \) at coverage level \( l \) is used to insure soybeans, \( ip_{o,i,j} \) is proceeds for crop \( o \) from crop insurance product \( i \) at coverage level \( j \), and \( q_{o,i,j} \) is the insurance premium for crop \( o \) under crop insurance product \( i \) at coverage level \( j \).

The Loan Deficiency and Marketing Loan programs make payments when prices are below loan rates. The function \( ldp_o \) captures these payments and equals:

\[
 ldp_o = y_o \cdot \max(0, p_{o,rate} - p_o)
\]

(15)

were \( p_{o,rate} \) is the loan rate. Counter-cyclical payments occur whenever the season average price, \( p_{o,sap} \) is below the trigger price, \( p_{o,tp} \). Per acre payments are made based on payment bushels, a quantity determined when base acres and yields were selected for the 2002 Farm Bill subject to an upper limit on per acre payments. Formally, counter-cyclical payments equal:
\[ cc_{p_o} = y_{o,bp} \cdot \max(0, \min(p_{o,lp} - p_{o,sap}, p_{o,l})) \]  

where \( y_{o,bp} \) is per acre program yield for crop \( o \) and \( p_{o,l} \) is the limit on the size of counter-cyclical payments.

Proceeds from the crop insurance are modeled for five crop insurance products representing the range of crop insurance products available in Illinois. The five products and their specific indemnity functions are:

1. Actual Production History (APH) insurance. APH is yield insurance that makes payments when a farm yield falls below a guaranteed level equal to a farmer-selected coverage level times the APH yield (usually based on a yield history from the unit). On a per acre basis, APH indemnities equal:

\[ ip_{o,aph,j} = p_{o,aph} \cdot \max(0, y_{o,aph} \cdot c_{o,aph,j} - y_{o}) \]  

where \( y_{o,aph} \) is the APH yield, \( p_{o,aph} \) is the indemnity price, and \( c_{o,aph,j} \) identifies the coverage level.

2. Revenue Assurance with the base price option (RA) insurance. RA makes payments when indemnified crop revenue falls below a guaranteed level. The guaranteed level equals the farmer-selected coverage level times the APH yield times the base price \( (p_{o,b}) \). Base prices are settlement prices for contracts during a spring period. Indemnified crop revenue equals yield times a harvest price \( (p_{o,h}) \). Harvest prices are settlement prices for futures contracts during an harvest period. On a per acre basis, indemnity payments equal:

\[ ip_{o,ra,j} = \max(0, p_{o,b} \cdot y_{o,aph} \cdot c_{o,ra,j} - p_{o,h} \cdot y_{o}) \]  

where \( c_{o,ra,j} \) identifies the coverage level.
3. Crop Revenue Coverage (CRC) insurance. CRC is a revenue insurance product that is similar to RA. The difference is that CRC uses the higher of the base or harvest price to determine guaranteed revenue:

\[
i_{p_{o,crc,j}} = \max(0, \max(p_{o,b}, \min(p_{o,h}, p_{o,b} + l)) \cdot y_{o,aph} \cdot c_{o,crc,j} - p_{o,h} \cdot y_o)
\]  

(19)

where \(c_{o,crc,j}\) identifies the coverage level and \(l\) is the limit increase.

4. Group Risk Plan (GRP) insurance. GRP is a county-level yield insurance that makes payments when county yield, \(y_{o,cy}\), falls below a guaranteed yield that equals a farmer-selected coverage level, \(c_{o,grp,j}\), times the expected county yield, \(y_{o,ecy}\):

\[
i_{p_{o,grp,j}} = \max(0, w_{o,grp} \cdot s_{grp} \cdot (y_{o,ecy} \cdot c_{o,grp,j} - y_{o,cy}) / (y_{o,ecy} \cdot c_{o,grp,j}))
\]  

(20)

where \(w_{o,grp}\) is a maximum protection level and \(s_{grp}\) is the percent of the coverage level that the farmer chooses.

5. Group Risk Income Plan (GRIP) insurance. GRIP makes payments when country revenue falls below a guaranteed revenue equal to the coverage level, \(c_{o,grip,j}\), times the base price times the expected county yield. On a per acre basis, payments equal:

\[
i_{p_{o,grip,j}} = \max(0, w_{o,grip} \cdot s_{grip} \cdot (p_{o,b} \cdot y_{o,ecy} \cdot c_{o,grip,j} - p_{o,h} \cdot y_{o,cy}) / (p_{o,b} \cdot y_{o,ecy} \cdot c_{o,grip,j}))
\]  

(21)

where \(w_{o,grip}\) is a maximum protection level and \(s_{grip}\) is the percent of the coverage level that the farmer chooses.

Gross revenue distributions are simulated for each insurance product at different coverage levels. For APH, RA, and CRC, three levels are simulated representing low (65 percent), medium (75 percent) and high (85 percent) coverage levels. The county-level products also are simulated at low (70 percent), medium (80 percent) and high (90 percent) coverage levels. The five products at three coverage levels plus a no-insurance option results in 16
insurance alternatives for each crop. All possible combinations of the insurance alternatives are allowed across both crops resulting in a total of 256 possible insurance alternatives (16 x 16).

**Simulation Parameters**

Random variables in the simulation model (futures prices, season average prices, county yields, and farm yields) and all other model parameters represent 2003 conditions. Yield distributions are modeled for an average farm in Logan County, Illinois. This county is a fairly typical one in Illinois.

**Prices.** In keeping with previous research, the futures price distributions are parameterized as lognormal distribution whose parameters were recovered using options market data. Futures and options data from the last trading day in February 2003 for the Chicago Board of Trade December corn and November soybean futures and options contracts were used to estimate the parameters of the futures price distribution using Black-Scholes option pricing methods. The resulting corn distribution has an expected value of $2.32 per bushel and a standard deviation of $0.43 per bushel. The soybean distribution has an expected value of $5.22 per bushel and a standard deviation of $.86 per bushel. Cash price ($p_o$) equals the futures price minus a fixed cash basis estimated using Illinois Agricultural Marketing Service (AMS) from 1999 through 2001. The basis is $.28 per bushel for corn and $.25 per bushel for soybeans.

Loan rates for 2003 in Logan county ($p_{o,rate}$) are $2.06 for corn and $5.16 for soybeans. Counter-cyclical payments are estimated using trigger prices ($p_{o,tp}$) of $2.32 for corn and $5.26 for soybeans, national loan rates ($p_{0,l}$) of $1.98 for corn and $5.00 for soybeans. Season average prices ($p_{o,sap}$) are estimated based on a function of futures prices using data from 1972 through 2002. Linear regressions were fit to yearly season average prices and settlement prices averaged
for the month of December for corn and November for soybeans. The resulting relationship for corn is $p_{c,sap} = .6315 + .7091f_c + e_c$ where $f_c$ is the average settlement price and $e_c$ is an error term with .1891 standard deviation. The t-statistic on the intercept is 11.3352. For soybeans, the relationship is $p_{s,sap} = 1.8725 + .6861f_s + e_s$ where $f_s$ is the average settlement price and $e_s$ is an error term with .4412 standard deviation. The t-statistic on the intercept is 10.3752.

**Yield distributions.** Based on work by Zanini, yield distributions were parameterized as Weibull distributions. National Agricultural Statistical Service (NASS) county yield series for 1972 through 2002 were detrended using a linear model and stated in terms of 2003 yields for purposes of representing current conditions. The Weibull distributions were fit to the detrended series using method-of-moments estimation. The resulting corn distribution has a mean of 157 bushels and standard deviation of 48 bushels. The county soybean distribution has a mean of 49 bushels and a standard deviation of 4.5 bushels.

The farm-level yield for a crop was set equal to the expected yield of Logan county’s fitted distribution. The county yield standard deviations were rescaled to reflect farm-level variability based on empirical evidence from Farm Business Farm Management (FBFM) record keeping system. To do so, farm series from the FBFM database with at least 12 years of data were detrended and fitted to a Weibull distribution. Then, for each farm, the ratio of farm standard deviation to county standard deviation was computed. To recover the final parameters of the yield distribution for the case farm, method of moments estimation was used subject to the constraint that their standard deviations equaled the county standard deviation times the average ratio of farm to county standard deviations. Resulting factors indicate that the standard deviation for the farm-level corn yield is 1.24 times the county standard deviation and the standard deviation for farm-level soybean yield is 1.47 times the county standard deviation.
**Correlations among random variables.** Farm to county yield correlations are calculated using the same set of FBFM farm data used to calibrate the farm yield distributions. The farm yield to futures price correlation is -.537 for corn and -.566 for soybeans. The correlation coefficients between the farm's corn yield and soybean yield is .780 and the correlation coefficient between the futures prices for corn and soybeans is .850.

**Insurance specifications.** The simulated revenue distributions require 2003 per acre premium costs for each of the insurance products at each coverage level. A basic unit option was used to generate premiums for APH, RABp, and CRC products. APH yields were set equal to the expected value for the farm yield. The resulting premiums were generated using the *iFARM* premium calculator (available at the *farmdoc* website at: [www.farmdoc.uiuc.edu/cropins/index.html](http://www.farmdoc.uiuc.edu/cropins/index.html)) and represented premiums farmers pay for insurance products after government subsidies have been subtracted. These premiums are not necessarily actuarially fair. The calculated premiums agree with quotes from the online premium quote software available at RMA’s website ([www.rma.usda.gov](http://www.rma.usda.gov)) as well.

Other simulation input variables include the indemnity price for APH products (*p_{aph}*), which is set to the 2003 maximum allowable level of $2.20 for corn and $5.40 for soybeans. The indemnity price for generating revenue guarantees (*p_{b}* equals $2.32 for corn and for 2002, and the maximum protection levels for GRP and GRIP are obtained from RMA ([www.rma.usda.gov](http://www.rma.usda.gov)). Protection levels are set at their maximums (*s_{grp} = s_{grip} = 1*).

**Simulation methods.** Revenue distributions were generated in Matlab using 10,000 iterations. The observations for the random variables were checked against the underlying relationships and found to be highly accurate, with simulated averages differing from their theoretical means by less than .001% on average, simulated standard deviations departing from
the underlying theoretical standard deviations by less than .01% on average, and simulated correlations differing from specified values by less than 3% on average.

**Empirical Determination of Efficient Sets**

Empirical cumulative distributions are constructed by ordering the observations from the lowest to highest revenue with each observation having $1/n$ probability. The $j$th ordered observation has a cumulative distribution function value of $j/n$ ($n = 10,000$). An empirical $VaR_a$ is equal to the revenue observation whose cumulative distribution function value is the closest to $a$. A $CVaR_a$ is found by averaging all revenues below and including the $VaR_a$ associated with the $CVaR_a$.

Efficient sets are found using the $VaR$, $CVaR$, and $SOSD$ criteria. For each criterion, pair-wise efficient-inefficient determinations are made for all 256 alternatives. An alternative is in the efficient set if it is not dominated by any of the other alternatives. $VaR$ and $CVaR$ efficient sets are determined using equations (7). SOSD efficient sets are determined using a discrete application of equation (12) in which alternative $x$ dominates $y$ when:

$$\sum_{j=1}^{k} x_j \geq \sum_{j=1}^{k} y_j \quad \text{for } k = 1, 2, 3, \ldots, n.$$  \hspace{1cm} (22)

where at least one inequality in (22) holds strictly. Levy details an algorithm which is used in this application that combines equation (22) with other SOSD conditions to increase computational speed.

**Results**

Descriptive statistics for the Logan County case farm's yield and revenue distributions are provided in Table 1. The expected value of gross revenue is $321 per acre, based on a 50-50
split between corn and soybeans. The $VaR_{0.01}$ for gross revenue is $204$ per acre, indicating that one percent of the time revenue will be less than $204$. The $CVaR_{0.01}$ is $185$, indicating that the expected value of the one percent lowest revenue cases is $185$ or, equivalently, that the conditional expected value of revenues below $204$ is $185$.

Statistics associated with the expected value and risk measures across the 256 insurance alternatives are shown in Table 2. The average of expected values across the 256 alternatives is $319$ per acre, ranging from a minimum of $315$ to a maximum of $326$. $CVaR_{0.01}$ values exhibit considerable range over the insurance alternatives. The minimum $CVaR_{0.01}$ is $185$ per acre, the maximum is $245$ per acre, giving a range of $60$ from the minimum to the maximum $CVaR_{0.01}$.

The range of $CVaR$ is larger for smaller $a$ values. For example, $CVaR_{0.01}$ has a range of $60$ across the insurance alternatives. This range is reduced to $32$ for $CVaR_{0.05}$, $22$ for $CVaR_{0.10}$, and $12$ for $CVaR_{0.25}$, indicating less difference for higher probabilities. A similar relationship exists for $VaR$s. The $VaR$ range is $43$, $17$, $12$, and $10$, respectively, for $a$ values of 1, 5, 10 and 25% (see Table 2).

The tradeoff between lower expected values and lower risks is most pronounced for low $a$. The correlation coefficient between the expected value and $CVaR_{0.01}$ is -.92 (see Table 2) indicating a strong risk-return tradeoff. This strong risk-return tradeoff is less pronounced and becomes non-existent for higher $a$. Correlation coefficients between the expected value and $CVaR$ are -.92, -.78, -.49, and .46 for $a$ of 1, 5, 10, and 25%, respectively (see Table 2). The larger tradeoffs at low $a$ and the strong risk-return correlations at low $a$ suggest focusing attention on low level of $a$ for a more meaningful discrimination among alternatives. Hence, $CVaR_{0.01}$ is used as the risk measure in the remainder of this paper.
**SOSD results.** The SOSD efficient set consists of 37 crop insurance alternatives, representing 14.5 percent total alternatives. Gloy and Baker (2002) found similar discriminatory power in their examination of insurance and hedging strategies in which SOSD and SOSD using a risk-free asset placed between 13 and 46 percent of the alternatives in the efficient set.

**Congruence of SOSD and CVaR Sets.** The efficacy of the CVaR and VaR measures is judged by how well CVaR and VaR efficient sets encompass the risk-return characteristics of the alternatives in the SOSD efficient set. Efficient sets developed using CVaR at \( a \) equal to 1, 5, 10, and 25% have 14, 11, 9, and 5 alternatives, respectively. Recall that, by definition, the CVaR efficient alternatives also are SOSD efficient.

Table 3 shows the 37 SOSD efficient sets sorted from low to high CVaR\(_{0.01}\). Expected values of the SOSD alternatives also are reported in Table 3. Given the strong negative correlation between the expected value and CVaR\(_{0.01}\) (i.e., -.92), the alternatives also are fairly well sorted from highest to lowest expected value.\(^5\) The first columns in Table 3 provide a clear representation of the risk-return tradeoffs across the alternatives, with higher expected values and higher risks in the top rows of the table and lower expected values and lower risks in the bottom rows of the table. Efficient sets are indicated by an "E" in the last five columns of the table. Over the risk-return range covered by alternatives in each efficient set, alternatives are fairly evenly distributed across the risk-return range.

Alternatives in the CVaR\(_{0.01}\) and CVaR\(_{0.05}\) efficient sets cover the same range of expected values and CVaR\(_{0.01}\) as are contained in the SOSD efficient set. This is illustrated in Table 4. The expected value range of the SOSD efficient set is from $315 to $326. The CVaR\(_{0.01}\) set a slightly smaller range in expected values from $317 to $326 while the CVaR\(_{0.05}\) set has exactly the same expected value range as the SOSD set. For the risk measure, CVaR\(_{0.01}\), efficient sets determined
using SOSD, $CVaR_{0.01}$, and $CVaR_{0.05}$ have exactly the same risk measure range of $204 \, CVaR_{0.01}$ up to $245 \, CVaR_{0.01}$. Therefore, the $CVaR_{0.01}$ and $CVaR_{0.05}$ efficient sets encompass the range of risk-return tradeoffs that exist in the SOSD efficient set. For this problem use of $CVaR$ at a of 1 and 5 percent will present a decision-maker with alternatives exhibiting a full range of risk-return characteristics.

Efficient sets at higher $a$ contain fewer alternatives and tend to eliminate the alternatives with the most risk reduction (see Table 4). The $CVaR_{0.10}$ efficient set has an alternative with a maximum $CVaR_{0.01}$ of $243$, down from the $245$ maximum for the $CVaR_{0.01}$ and $CVaR_{0.05}$ efficient sets. The $CVaR_{0.25}$ efficient set has a maximum of $230$. For this insurance problem, $CVaR$ efficient sets with higher $a$s do not include alternatives that have the highest $CVaR_{0.01}$. This occurs because $CVaRs$ with higher $a$ become more like the expected values and hence become more positively correlated with the expected value. Increasing a is similar to putting more weight on the expected value and less weight on risk.

**Congruence of SOSD and VaR Sets.** For this empirical problem, $VaR$ efficient sets never contain more alternatives than do $CVaR$ efficient sets at the same $a$ level. $VaR$ efficient sets developed using $a$ equal to 1, 5, 10, and 25% contain 11, 6, 5, and 4 alternatives, respectively (see Table 4). At the 1, 5, 10 and 25% levels, the $VaR$ efficient set has 3, 5, 4, and 1 alternative less than the $CVaR$ efficient set. The percentage reduction made by using $VaR$ versus $CVaR$ is large. The $VaR$ set is 21% smaller than the $CVaR$ set at a equal to 1%, 45% smaller at a equal to 1%, 44% smaller at a equal to 10%, and 20% smaller at a equal to 25%.

As $a$ increases, the $VaR$ efficient set eliminates lower risk alternatives compared the $CVaR$ efficient set. The maximum $CVaR_{0.01}$ is $230$ for the $VaR_{0.05}$ efficient set compared to $245$ for the $CVaR_{0.05}$ efficient set. At a equal to 10, the maximum $CVaR_{0.01}$ is $215$ for $VaR$ compared
to $243 for $CVaR$ efficient set. At a equal to 25, the maximum $CVaR_{0.1}$ is $215 compared to $230. As suggested by conceptual studies, $VaR$ eliminates lower risk alternatives from consideration when, in fact, they should not be eliminated.

**Summary and Conclusions**

This study compares Value-at-Risk ($VaR$) and Conditional Value-at-Risk ($CVaR$) for use as risk measures when evaluating crop insurance alternatives. While popular, the $VaR$ risk measure has known problems. For example, rankings made using $VaR$ are not necessarily consistent with a rational utility function (Kaplanski and Kroll) and use of $VaR$ may cause insurances alternatives to be eliminated from consideration when they should not be eliminated (Basak and Shapiro). As a result of these problems the $CVaR$ risk measure has been developed.

$VaR$ and $CVaR$ use is illustrated in a conceptual framework in which a crop revenue insurance alternative is compared to a no insurance alternative. By design, the revenue insurance truncates the revenue distribution and is actuarially fair and thus will have the same expected value as the no insurance alternative. The revenue insurance alternative is preferred by all risk averse individuals who maximize the expected value of a continuous, concave utility function. The $VaR$ risk measure will not always indicate that revenue insurance is the least risky alternative. Hence, use of $VaR$ can lead to the elimination of the revenue insurance even though the revenue insurance is preferred by all risk averse individuals. $CVaR$ mitigates this problem.

Linkages are made between efficient sets developed using second order stochastic dominance (SOSD) and efficient sets developed using $VaR$ and $CVaR$ in a risk-return framework. $CVaR$ efficient sets are shown to be subsets of the SOSD efficient set, while $VaR$ efficient sets are not necessarily subsets of the SOSD efficient set. As a result, the $CVaR$
efficient set will not contain alternatives that any risk averse individual would not consider – importantly, the same can not be said for \( VaR \) use.

\( VaR \) and \( CVaR \) use are compared in an evaluation of crop insurance alternatives on a central-Illinois farm raising corn and soybeans. For this case, efficient sets are determined using \( VaR \) and \( CVaR \) as risk measures for different levels of \( a \). This evaluation shows that the \( VaR \) efficient set has a smaller number of alternatives in its efficient set compared to the \( CVaR \) efficient set for a given \( a \)-level. Moreover, and more damaging, the \( VaR \) risk measure tends to eliminate the alternatives that have the most reduction in downside risk when compared to \( CVaR \) or SOSD efficient sets. Hence, use of a \( VaR \) risk measure to rank alternatives will tend to prematurely eliminate the use of crop insurance alternatives. Therefore, based on both conceptual and empirical considerations, it is concluded that \( CVaR \) should be used in crop insurance evaluations rather than \( VaR \) risk measures.

The empirical evaluation has been limited to crop insurance alternatives and, therefore, may not generalize to all other applications. However, there are many applications within agriculture that have similar characteristics to the crop insurance problem. \( VaR \) use will be problematic in any application in which distributions are truncated or the distributions are not symmetric for some other reasons. Applications with these characteristics include evaluations of grain or livestock marketing alternatives that include options contracts and financial analysis of firms in which bankruptcy or default is a possibility. Future research should evaluate \( VaR \) and \( CVaR \) use in these applications.

This research only compared \( VaR \) and \( CVaR \) use. There are good reasons to believe that \( CVaR \) will outperform most other risk measures in many applications (Artzner, et.al.). For example, some individuals use the probability below a benchmark return as a risk measure. This
risk measure will have the same problems as VaR does. Others use standard deviation or variance as a measure of risk. These measures have problems ranking alternatives when distributions are not symmetric. Further research should compare a broader set of risk measures for evaluating risky alternatives.
Footnotes

1 VaR also has been defined relative to the distribution of ending period of wealth and net returns. The focus of the empirical application is on revenue; hence, definitions are made relative to revenue.

2 VaR can also be defined relative to some reference point such as the mean (x - VaR, where x is the reference point). General results do not depend on the definition.

3 The specific alternative modeled is Revenue Assurance with a base price option.

4 These problems exist for most popular risk measures including probability below a benchmark and Fishburn risk measures. Standard deviations and variances also can have problems ranking alternatives when distributions are not symmetric.

5 The risk-return tradeoff would be exact if only efficient CVaR₀₁ alternatives are included in Table 3. By construction, the CVaR₀₁ criteria eliminates alternatives the have a lower expected value for a given CVaR₀₁.
References


Figure 1. Cumulative Distribution Functions for Revenue with and without Revenue Insurance.

\[ a^* = F_{NI}(r^* - q) \]
Table 1. Means, VaRs, and CVaRs for Logan County Case Farm (2003).

<table>
<thead>
<tr>
<th></th>
<th>Yield (Bu. per acre)</th>
<th>Gross Revenue¹</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Corn</td>
<td>Soybeans</td>
<td></td>
</tr>
<tr>
<td>Expected Value</td>
<td>159</td>
<td>49</td>
<td>$321</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>27</td>
<td>7</td>
<td>$46</td>
</tr>
<tr>
<td>Value-at-Risk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>87</td>
<td>30</td>
<td>$204</td>
</tr>
<tr>
<td>5%</td>
<td>109</td>
<td>36</td>
<td>$242</td>
</tr>
<tr>
<td>10%</td>
<td>121</td>
<td>40</td>
<td>$261</td>
</tr>
<tr>
<td>25%</td>
<td>141</td>
<td>45</td>
<td>$292</td>
</tr>
<tr>
<td>Conditional Value-at-Risk</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>75</td>
<td>26</td>
<td>$185</td>
</tr>
<tr>
<td>5%</td>
<td>95</td>
<td>32</td>
<td>$218</td>
</tr>
<tr>
<td>10%</td>
<td>106</td>
<td>35</td>
<td>$235</td>
</tr>
<tr>
<td>25%</td>
<td>122</td>
<td>39</td>
<td>$261</td>
</tr>
</tbody>
</table>

¹ For an acre with 50% corn and 50% soybeans.
Table 2. Statistics and Correlation Coefficients Across 256 Insurance Alternatives, Logan County Farm (2003).

<table>
<thead>
<tr>
<th>Statistic(^1)</th>
<th>Mean</th>
<th>Min(^2)</th>
<th>Max(^3)</th>
<th>Range(^4)</th>
<th>Expected Value and Risk Measures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$ per Acre</td>
<td>EV(^5)</td>
<td>CVaR(_{0.01})</td>
<td>CVaR(_{0.05})</td>
<td>CVaR(_{0.1})</td>
</tr>
<tr>
<td>Expected Value</td>
<td>319</td>
<td>315</td>
<td>326</td>
<td>12</td>
<td>1.00</td>
</tr>
<tr>
<td>CVaR(_{0.01})</td>
<td>214</td>
<td>185</td>
<td>245</td>
<td>60</td>
<td>-0.92</td>
</tr>
<tr>
<td>CVaR(_{0.05})</td>
<td>233</td>
<td>218</td>
<td>251</td>
<td>32</td>
<td>-0.78</td>
</tr>
<tr>
<td>CVaR(_{0.1})</td>
<td>245</td>
<td>235</td>
<td>257</td>
<td>22</td>
<td>-0.49</td>
</tr>
<tr>
<td>CVaR(_{0.25})</td>
<td>265</td>
<td>260</td>
<td>272</td>
<td>12</td>
<td>0.46</td>
</tr>
<tr>
<td>VaR(_{0.01})</td>
<td>223</td>
<td>204</td>
<td>247</td>
<td>43</td>
<td>-0.86</td>
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<tr>
<td>VaR(_{0.05})</td>
<td>248</td>
<td>241</td>
<td>259</td>
<td>17</td>
<td>-0.07</td>
</tr>
<tr>
<td>VaR(_{0.1})</td>
<td>263</td>
<td>259</td>
<td>271</td>
<td>12</td>
<td>0.62</td>
</tr>
<tr>
<td>VaR(_{0.25})</td>
<td>290</td>
<td>286</td>
<td>297</td>
<td>10</td>
<td>0.95</td>
</tr>
</tbody>
</table>

1 Statistics are given across the 256 insurance alternatives.
2 Minimum.
3 Maximum.
4 Equals the maximum minus the minimum.
5 Expected value.
6 CVaR stands for conditional value-at-risk.
7 VaR stands for value-at-risk.
Table 3. CVaR Efficient Alternatives, Logan County Case Farm (2003).

<table>
<thead>
<tr>
<th>Efficient by Second Order Stochastic Dominance</th>
<th>Expected Value ($ per acre)</th>
<th>Efficient Sets by CVAR&lt;sub&gt;01&lt;/sub&gt;, CVAR&lt;sub&gt;05&lt;/sub&gt;, CVAR&lt;sub&gt;10&lt;/sub&gt;, CVAR&lt;sub&gt;25&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Policy for</strong>:</td>
<td>Corn Soybeans CVAR&lt;sub&gt;01&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>GRIP-90</td>
<td>GRIP-90 326 204</td>
<td>E&lt;sup&gt;4&lt;/sup&gt; E E E E</td>
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<tr>
<td>GRIP-90</td>
<td>none 325 204</td>
<td>E</td>
</tr>
<tr>
<td>GRIP-90</td>
<td>GRIP-80 325 204</td>
<td>E</td>
</tr>
<tr>
<td>GRIP-90</td>
<td>APH-65 324 208</td>
<td></td>
</tr>
<tr>
<td>GRIP-90</td>
<td>GRP-90 325 210</td>
<td>E E E E E</td>
</tr>
<tr>
<td>GRP-90</td>
<td>none 324 213</td>
<td></td>
</tr>
<tr>
<td>GRP-90</td>
<td>GRP-80 323 214</td>
<td></td>
</tr>
<tr>
<td>GRP-90</td>
<td>GRP-90 324 214</td>
<td></td>
</tr>
<tr>
<td>GRP-90</td>
<td>GRP-90 323 214</td>
<td></td>
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<tr>
<td>GRP-90</td>
<td>GRP-90 325 215</td>
<td>E E E E E</td>
</tr>
<tr>
<td>GRP-90</td>
<td>APH-75 324 216</td>
<td>E</td>
</tr>
<tr>
<td>GRP-90</td>
<td>RA-65 323 216</td>
<td></td>
</tr>
<tr>
<td>GRP-90</td>
<td>APH-65 323 216</td>
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<tr>
<td>GRP-90</td>
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<td>E</td>
</tr>
<tr>
<td>GRP-90</td>
<td>RA-75 322 221</td>
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<tr>
<td>GRP-90</td>
<td>APH-75 323 222</td>
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</tr>
<tr>
<td>APH-75</td>
<td>APH-75 319 222</td>
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<td>GRP-90</td>
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<td>GRIP-90</td>
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<td>E E E E</td>
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<td>RA-75</td>
<td>APH-75 319 224</td>
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<td>GRIP-90</td>
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<td>GRP-90</td>
<td>RA-85 321 227</td>
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<td>APH-85 322 229</td>
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<td>CRC-85</td>
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<tr>
<td>CRC-85</td>
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<td>E</td>
</tr>
<tr>
<td>CRC-85</td>
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<td>RA-85</td>
<td>APH-75 318 237</td>
<td>E</td>
</tr>
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<td>RA-85</td>
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</tr>
<tr>
<td>CRC-85</td>
<td>APH-85 315 243</td>
<td>E E</td>
</tr>
<tr>
<td>RA-85</td>
<td>CRC-85 316 244</td>
<td></td>
</tr>
<tr>
<td>RA-85</td>
<td>APH-85 317 245</td>
<td>E E</td>
</tr>
</tbody>
</table>

1 APH is Actual Production History, RA is Revenue Assurance, CRC is Crop Revenue Coverage, GRP is Group Risk Plan and GRIP is Group Risk Plan. Numbers indicate coverage levels.

2 CVaR indicates Conditional Value-at-Risk.

4 E indicates that the alternative is CVaR efficient.
Table 4. Return and Risk Characteristics of Alternatives in Different Efficient Sets, Logan County Case Farm (2003).

<table>
<thead>
<tr>
<th>Efficient Set by:</th>
<th>Alternatives in Set</th>
<th>Expected Value Range $ per acre</th>
<th>CVaR_{0.01} Range $ per acre</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No.</td>
<td>Min¹</td>
<td>Max²</td>
</tr>
<tr>
<td>SOSD</td>
<td>37</td>
<td>315</td>
<td>326</td>
</tr>
<tr>
<td>CVaR₀₁</td>
<td>14</td>
<td>317</td>
<td>326</td>
</tr>
<tr>
<td>CVaR₀₅</td>
<td>11</td>
<td>315</td>
<td>326</td>
</tr>
<tr>
<td>CVaR₁₀</td>
<td>9</td>
<td>315</td>
<td>326</td>
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<tr>
<td>CVaR₂₅</td>
<td>5</td>
<td>321</td>
<td>326</td>
</tr>
<tr>
<td>VaR₀₁</td>
<td>11</td>
<td>317</td>
<td>326</td>
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<tr>
<td>VaR₀₅</td>
<td>6</td>
<td>321</td>
<td>326</td>
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<tr>
<td>VaR₁₀</td>
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<td>323</td>
<td>326</td>
</tr>
<tr>
<td>VaR₂₅</td>
<td>4</td>
<td>325</td>
<td>326</td>
</tr>
</tbody>
</table>

¹Gives the minimum value for an alternative in the respective set.
²Gives the maximum value for an alternative in the respective set.
³Equals the maximum minus the minimum.